FLUID MECHANICS

Fluid Mechanics

Module I — 1. Introduction and Fundamental Concepts
2. Fluid Statics

Module II — 3. Basic equations in Integral form for a control volume
4. Differential analysis to Fluid Motion

Module III — 5. Incompressible Inviscid flow

Module IV — 6. Flow through pipes (Incompressible flow)
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What is a fluid ?

A substance that deform continuously ~~the~~ when acted on by a shearing stress of ~~deform~~ any size.

couple of key words :- deforms, continuous
however a solid can also deform.

one of the key deference is continuously deform. when a shearing stress acting on it

you know what is a stress is
Force per unit area that is how we define a stress is

what is a shearing stress?
If it is not shearing which other types of stress could we be having.
tensile or normal stress

what is a shearing stress then how do you define it very easily.

tensile ~~Normal~~ → pulling or pushing
(-ve) (tensile) (+ve) (compressive).

parallel to the plan of the area,
A shearing stress on a surface is parallel to the surface, where as normal stress is perpendicular to the surface, it could be tensile if pulling it could be compressive if pushing
Here the one that we really care about is the shearing stress.

so fluid will continuously deform as long as there is a shearing stress.
The final key word is any size

there is a reason for that last part of sentence.

Any size is replace by
shearing stress does not matter however small it is

If we make shear stress smaller and smaller
we see the deformation is continuous.

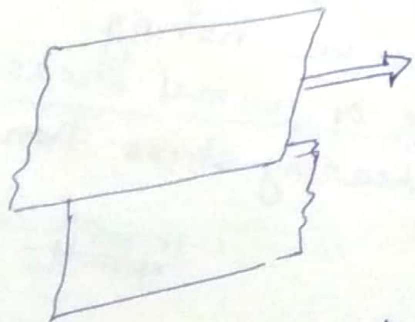
if it is a fluid it does not matter how small it is

Mathematically we say even if it infinitidecimal shearing stress acting on that surface, the fluid will continue to deform.

that why we say of any size

$$\text{stress} = \frac{\text{force}}{\text{area}}$$

A simple experiment we can think of
I can put a fluid between two ~~pe~~ plates



→ Flat channel
→ two parallel plate

I keep one fixed, I put the fluid between two plates and move the top one.
we can see an arrow → it is moving to the right

I want to shear that fluid
the fluid in this gap will move

there is some minor exception to this definition of fluid.
there is some particular type of fluid behave little bit differently, we will talk about latter

→ ~~upper plate is moving to the right~~
Think that we are trying to solve a fluid mechanics problem.
→ we

Basic equations

Solving problems in fluid mechanics → storm in the weather

There is a fluid mechanics problem that I want to solve

(then we can ask ourself) what do I look for solving / what do I want my fluid ~~mechanics~~ to satisfy.

Must satisfy — in terms of physics or mathematics of that fluid

→ these are laws we can almost uniformly think of in any branch of physics or engineering.

(bottle 90% water gone) → Consideration of mass — that is one thing we need to account for.

1. [conservation of mass] → the fluid disappear at some place may be somewhere else.

Is momentum always conserved
→ linear momentum

2. [Newtons second law] ← conservation of momentum equation

newton second law

Let say → Linear momentum of a system is conserved if there are no friction
no external forces acting on the system

refer to equation that come out of newton second law.

when we are solving ~~wat~~ What else we should worry about when we are solving a fluid mechanics problem. (2a) Principle of angular momentum.

what about energy

3. [1st Law of thermodynamics] ← conservation of energy.

(name for conservation of energy) ↗

4. [2nd law of thermodynamics] * — since we know about it.

This is a very general picture, but there are many details we have to worry about to for solving a ~~real~~ actual fluid mechanics problems.

we have a fluid mechanics problem we worry about
→ conservation mass
→ newton second law
→ 1st law of thermodynamics

* → Not all basic laws are always required to solve any one problems
* — On the other hand, in many problems it is necessary to
*→ bring into the analysis additional relations the describe the behaviour of physical properties of fluids
→ ~~fluid properties~~. ideal gas equation of state

$$p = \rho R T$$

$P = \rho RT$

→ is a model that relates density to pressure and temperature for many gases under normal conditions.

Method of Analysis

→ The first step in solving a problem is to define the system that you are attempting to analyze.

→ In basic mechanics, we made extensive use of the free body diagram

→ We will use a system or control volume, depending on the problem being studied.
(These concept are identical to that we use in thermodynamics) open system or closed system

→ we can use either one to get mathematical expressions for each of the basic laws.

→ In thermodynamics they were mostly used to obtain expression for conservation of mass, the first and second law of thermodynamics.

→ In our study of fluid mechanics, we will be most interested in conservation of mass and Newton's second law of motion

→ In thermodynamics our focus was energy in fluid mechanics it will mainly be forces and motion

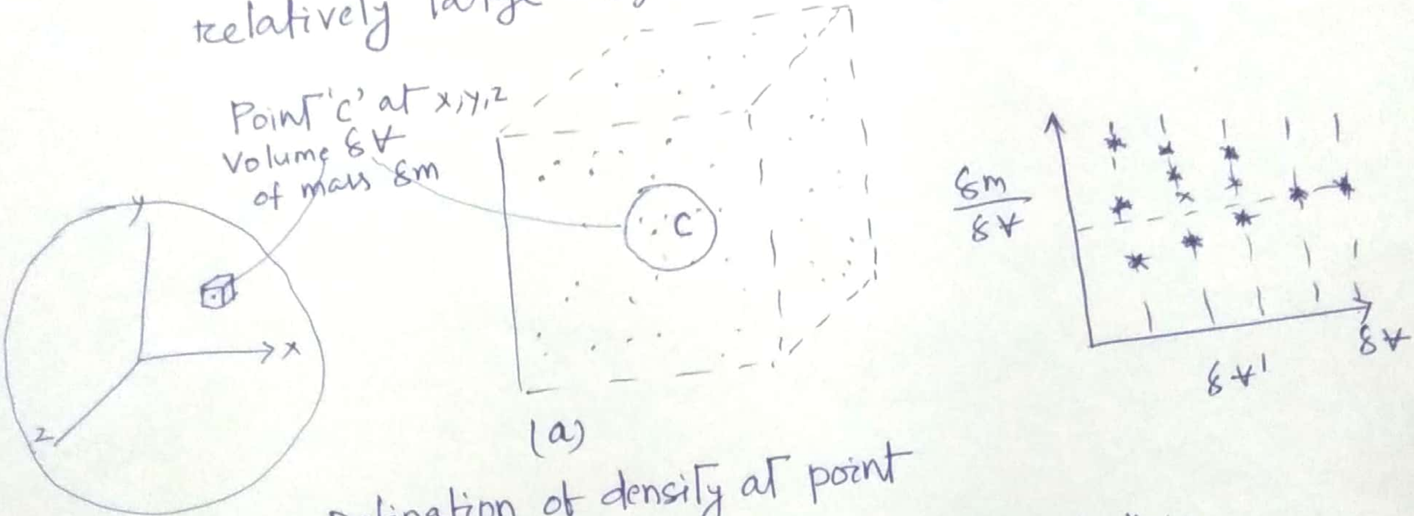
→ we must always be aware of whether we are using a system or a control volume approach because each leads to different mathematical expressions of these laws.

Fluid as Continuum

→ We are familiar with fluids — the most common being air and water — and we experience them as being 'smooth' (i.e) as being continuous medium.

→ Unless we use specialized equipment, we are not aware of the underlying molecular nature of fluids.

→ This molecular structure is one in which the mass is not continuously distributed in space, but is concentrated in molecules that are separated by relatively large regions of empty space.



(a)

Defination of density at point
Fig(a)

Fig (a) shows a schematic representation of this

⇒ A region of space "filled" by a stationary fluid (e.g. air, treated as a single gas) looks like a continuous medium, but if we zoom in on a very small cube of it, we can see that we mostly have empty space, with gas molecule scattered around, moving at high speed (indicated by high temperature)

⇒ Note that the size of the gas molecules is greatly exaggerated (they would be almost invisible even at this scale) and that we have placed velocity vectors only on a small sample.

⇒ We wish to ask: What is the minimum volume δV that a 'point' c must be, so that we talk about continuous fluid properties such as ~~tha~~ the density at a point?

⇒ In other words, under what circumstances can a fluid be treated as a continuum, for which, by definition, properties vary smoothly from point to point?

⇒ This is an important question because the concept of a continuum is the basis of classical fluid mechanics.

⇒ Consider how we determine the density at a point. Density is defined as mass per unit volume

In fig (a) the mass δm will be given by the instantaneous number of molecules in δV (and the mass of each molecule), so the average density in volume δV is given by

$$\rho = \frac{\delta m}{\delta V}$$

⇒ We say "average" because te number of molecules in δV and hence te density, fluctuates

⇒ For example, if the gas in fig (a) was air at standard temperature and pressure (STP) and the volume δV was a sphere of diameter 0.01 μm,

there might be 15 molecules in δV

but an instant later there might be 17 (three might enter while one leaves)

Hence, the density at "point" c randomly fluctuates in time, as shown in fig b

⇒ In this figure each vertical dashed line represents a specific chosen volume, δV, and each data point represents the measured density at an instant

⇒ For very small volumes, the density varies greatly, but above a certain volume, δV, the density becomes stable — the volume now encloses a huge number of molecules.

For example, if δV = .001 mm3 (about te size of a grain of sand)

there will on average 2.5×10^{13} molecules present.

For example, we now have a workable defination of density at a point

$$\rho = \lim_{\delta V \to \delta V'} \frac{\delta m}{\delta V} \quad \text{--- (1)}$$

Since point 'C' was arbitrary, the density at any other point in the fluid could be determined in the same manner.

⇒ If density was measured simultaneously at an infinite number of points in the fluid, we would obtain an expression for the density distribution as a function of the space co-ordinates

$$\rho = \rho(x, y, z) \text{ at a given instant}$$

⇒ The density at a point may also vary with the time (as a result of work done or by fluid and/or heat transfer to the fluid).

⇒ Thus the complete representation of the density (the field representation) is given by

$$\rho = \rho(x, y, z, t) \quad \text{--- (2)}$$

Since density is a scalar quantity, requiring only the specification of a magnitude for a complete description, the field represented by eqn (2) is a scalar field.

⇒ An alternative way of expressing the density of a substance (solid or fluid) is to compare is to an accepted reference value, typically the maximum density of water ρ_{H_2O} (1000 kg/m³ at 4°C (277 K). Thus the specific gravity, SG, of a substance is expressed as

$$SG = \frac{\rho}{\rho_{H_2O}} \quad \text{— (3)}$$

For example, the SG of mercury is typically 13.6 – (mercury is 13.6 times as dense as water)

The S.G of a liquids is a function of temperature; for most liquids specific gravity decreases with increasing temperature.

⇒ The specific weight 'γ' of a substance is another useful material property. It is defined as the weight of a substance per unit volume and given as

$$\gamma = \frac{mg}{V} \rightarrow \boxed{\gamma = \rho g} \quad \text{— (4)}$$

For example, specific weight of water is approximately 9.81 KN/m³.

Physical properties of fluid

We will discus another important fluid mechanics, the property of viscosity.

We are aware of viscosity from our everyday experience

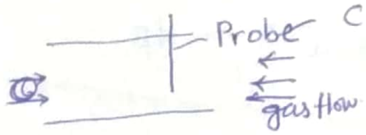
video – fluid flow with different viscosity
water, ~~gelatin~~ silicon

Before Talking about viscosity I want to bring to your attention the No-slip condition

Viscosity

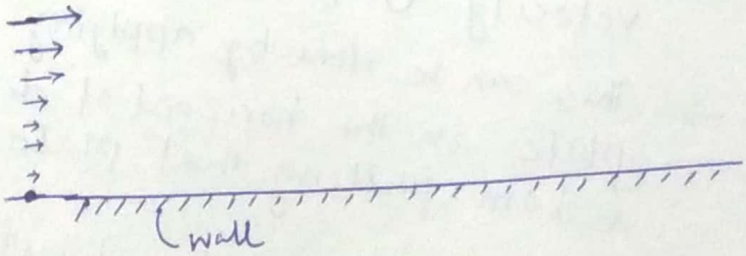
1. No-slip Condition

example :-



can inject Marker gas dye (color)
we can raise move the probe vertically
→ At the buttom the fluid seems to be stuck

This shows the notion of no-slip condition.



1. Fluid is moving from left to right
2. over the surface of the wall
3. we shaw that the fluid is moving with significant velocity at a point away from the wall
4. closer to the bottom wall, velocity is lower and infact on the surface the fluid is not moving

→ Let us see what happen in infinitdesimal time 'dt'

In 'dt' the upper plate will move a distance

$dx = U\,dt$ — dt → small time

↑ displacement of the upper plate

→ the upper plate displace a distance 'dx' we can measure the angle $d\beta$

→ the tangent of the angle $d\beta$ is given by

$\tan d\beta = \frac{dx}{h} \sim d\beta$ — h → distance of separation of the plate is

This is a small angle so $\tan d\beta$ is approximated as $d\beta$

If I look at the time variation of '$d\beta$'

$$\frac{d\beta}{dt} = \frac{1}{h}\frac{dx}{dt}$$

Since dx is the displacement

So $$\frac{d\beta}{dt} = \frac{1}{h}\frac{dx}{dt} = \frac{U}{h}$$

→ The rate at which angle is changing is ratio of velocity 'U' and the separation distance 'h'

→ we can approximate this ~~variation~~ by assuming the variation to be linear as

$$\frac{U}{h} = \frac{du}{dy}$$

→ assuming y co-ordinate, the velocity at that point is 'u'

then the approximation is

$$\frac{U}{h} = \frac{du}{dy}$$

→ This is an important quantity in fluid mechanics

→ we denote it

<u>Shear strain rate</u>

→ we can measure the shear strain rate

→ so shear strain rate is the velocity gradient in the vertical direction.

$$\text{Units} = \frac{du}{dy} \quad \frac{[m/s]}{m}$$

$$= \left[\frac{1}{s}\right]$$

→ what we do next is relate the shear strain rate and the force.

→ The force I apply at the top plate is F

F → shear strain rate

→ make connection between tc two

$$F \rightarrow \frac{\partial u}{\partial y}$$

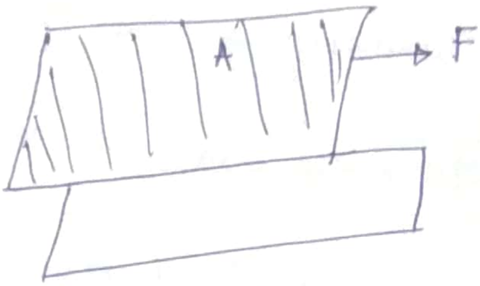
→ Instead of relating to the force we relate to the shear stress

→ we define the shear stress

Shear stress = $\frac{F}{A}$

A → Area of the plate

→ In 3D, I have a plate at the top, and another plate at bottom, moving the top plate by applying force



shear stress = $\frac{F}{A} = \tau$ (Tao)

$$\tau \rightarrow \frac{\partial u}{\partial y}$$

relationship between stress and strain rate

→ It ~~show~~ so happen that there is a particular type of fluid for which.

It $\tau = G \frac{\partial u}{\partial y}$

we call it a Newtonion fluid

when ~~that~~ relationship is linear.

This constant of propertionality we call it precisely viscosity..

This connect the shear stress and shear strain rate.

C = viscosity = μ (mu)

Newtonion fluid

τ

$$\tau = \mu \frac{\partial u}{\partial y}$$

we constrain our discussion to simple geometry
This is for a simple geometry, this relationship for complex geometry will be much more complicated.

* ~~we this reaction we~~ can tackle problems.

Units of viscosity (dynamic viscosity) μ.

$$[\mu] = \frac{[\tau]}{\left[\frac{\partial u}{\partial y}\right]} = \frac{[N/m^2]}{\left[\frac{m/s}{m}\right]} = \frac{N \cdot s}{m^2} = \mu$$

$$N = kg \frac{m}{s^2}$$

equivalent

$$[\mu] = \frac{kg}{m \cdot s}$$

Another viscosity

Kinematic viscosity ← time

(nu) $$\nu = \frac{\mu}{\rho}$$

units $$[\nu] = \frac{\frac{kg}{m \cdot s}}{\frac{kg}{m^3}} = \left\{\frac{m^2}{s}\right\}$$

Poise

Standard value

	Air	Water	Oil
$\mu \left[\frac{kg}{m \cdot s}\right]$	1.8×10^{-5}	1.1×10^{-3}	0.38

↓ (water) at 20°c is 0.01 poise or 1 centipoise.

Other units of μ

In MKS (matric gravitational system) = $\frac{kgf - sec}{m^2}$

(cgs units.) In metric absolute system

$\mu = \frac{dyne - sec}{m^2}$

$\frac{g}{cm \cdot s}$ = poise = P (classical unit)

$cp = 10^{-2} P$

this is also called poise (after Poiseuille)

Centipoise is one hundredth of a poise (1 centipoise = $\frac{1}{100}$ poise.)

$1 \frac{N \cdot s}{m^2} = 10$ poise.

Newtonion fluid — Air & water

1) effect of temp/pressure on viscosity

2) No slip condition

3) compressibility

4) vapour pressure

1) Effect of ~~viscosity~~ Pressure on viscosity
→ Pressure has a moderate effect on viscosity
• viscosity of gases & most liquid increases slowly with pressure.
eg:- For water change in viscosity is only a few percent upto 100 atm.
So we can neglect pressure effect.

ii) Effect of Temperature on viscosity.
→ Temperature has a strong effect on viscosity

μ → cohesive force & molecular momentum transfer
→ inter molecular force of cohesion
→ molecular momentum exchange.

Liquid $\mu \downarrow$ $T \uparrow$
(predominant)
cohesive force > molecular momentum transfer;
molecules closely packed → $T\uparrow$ C.F↓ hence $\mu\downarrow$

gas $\mu \uparrow$ $T \uparrow$
cohesive force < molecular momentum transfer (predominant)
molecules loosely packed → $T\uparrow$ M.M.T.↑ hence $\mu\uparrow$

<u>Relation</u> between μ & T

<u>For liquids,</u>

$$\mu = \mu_0 \left(\frac{1}{1 + \alpha t + \beta t^2} \right)$$

where μ = viscosity of liquid at t°C, in poise
μ_0 = " " " at 0°C, in poise
α, β → Constant for liquid

For water $\mu_0 = 1.79 \times 10^{-3}$ poise.
$\alpha = 0.03368$, $\beta = 0.000221$

(Temp) $t\uparrow$, $\mu\downarrow$

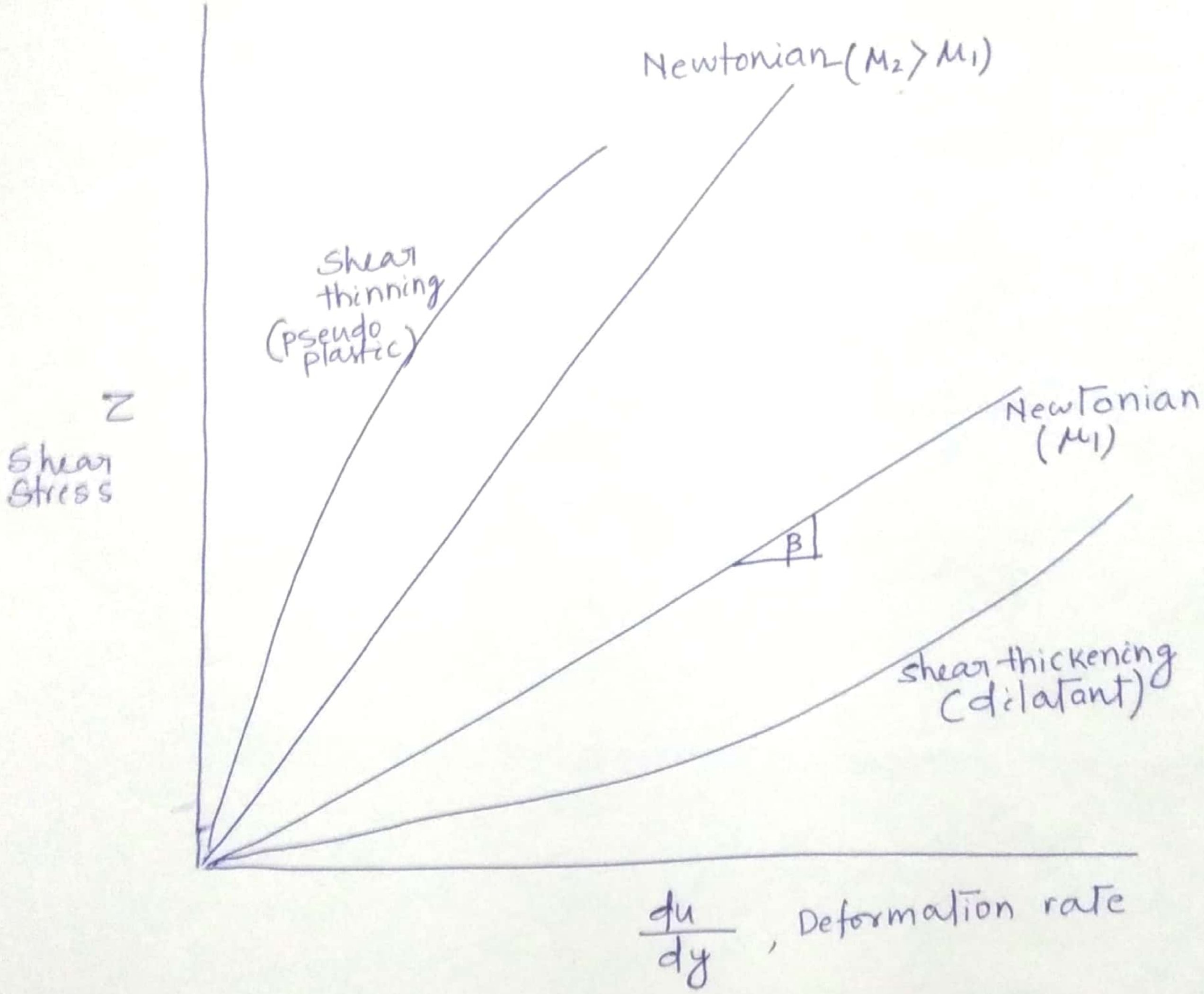
<u>For gas</u>

$$\mu = \mu_0 + \alpha t - \beta t^2$$

For air $\mu_0 = 0.000017$, $\alpha = 0.00000056$
$\beta = 0.1189 \times 10^{-9}$

(temp) $t\uparrow$, $\mu\uparrow$

Let us plot the variation of shear stress (τ) with respect to strain rate/deformation rate ($\frac{du}{dy}$)
(Rheological diagram)



→ Fluid in which shear stress ~~are~~ is not directly proportional to deformation rate are non-Newtonian.

example :- many common fluid exhibit non-Newtonian behavior.
→ toothpaste, starch (in water)

→ Tooth-paste behave as a fluid when squeezed from the tube (However it does not run out itself when the cap is removed)

→ There is a threeshold or yield stress below which ~~toothpaste~~ behave as solid

[our ~~def~~ Defination of fluid is valid for substances that have zero yield stress]

* Non-Newtonian fluid commonly are classified as having time independent or time-dependent behavior.

→ The relationship between shear stress (τ) and strainrate $\left(\frac{du}{dy}\right)$ for time-independent fluids is given by power law model. (for 1-D flow)

$$\tau = K\left(\frac{du}{dy}\right)^n$$

Here, the exponent 'n' is called the flow behavior index,
the coefficient 'K' is called consistency index

this eqn reduces to (

$$\tau = K \left|\frac{du}{dy}\right|^{n-1} \frac{du}{dy}$$

$$= \eta \frac{du}{dy}$$

when $n = 1$, $K = \mu$
→ variation is linear (Newtonion fluid) ~~flowion~~

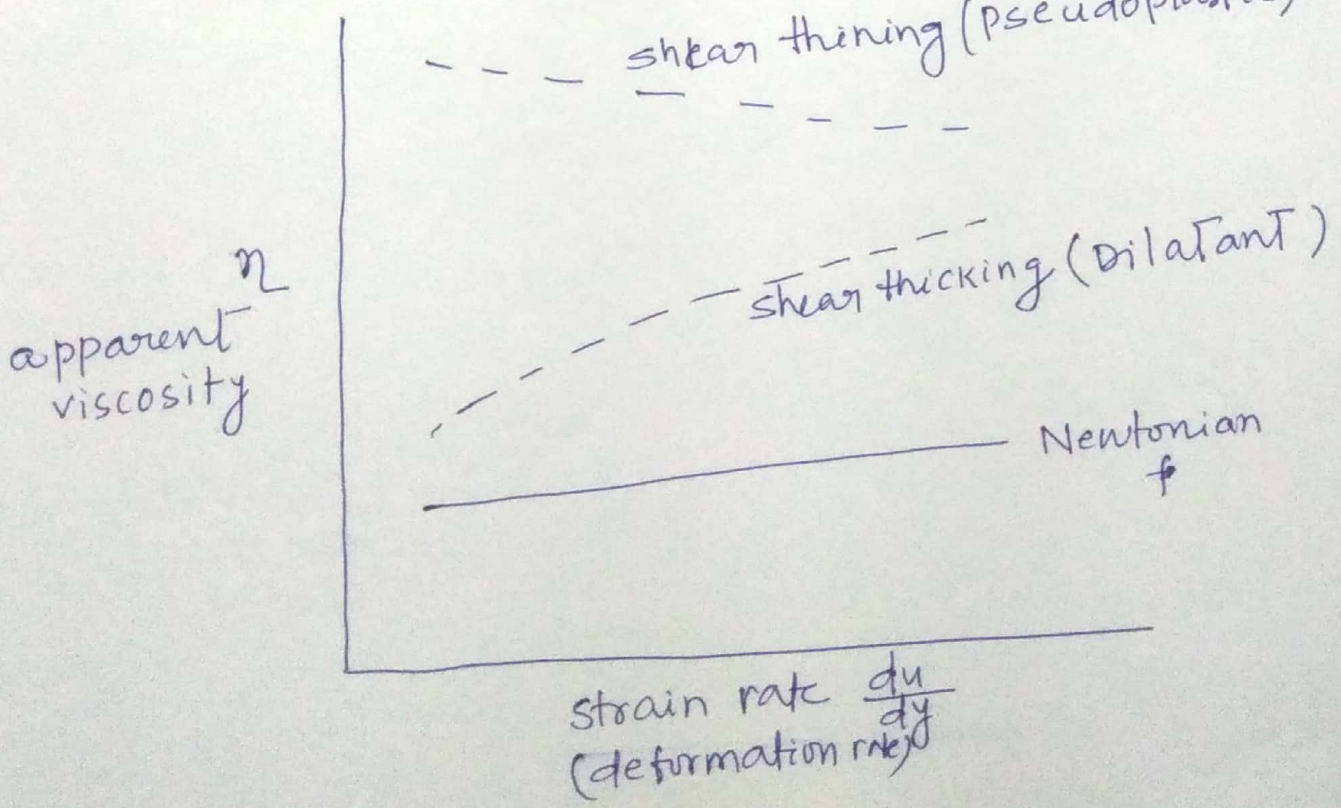
the term $\eta = K \left|\frac{du}{dy}\right|^{n-1}$ is called apparent viscosity

Difference between μ (Newtonion viscosity) & η (apparent viscosity)

→ μ is constant (except for temperature effects)

→ η depends upon the shear rate

⇒ Most non-Newtonian fluids have apparent viscosities that are relatively high compared with the viscosity of water.



⇒ Fluids in which apparent viscosity decreases with increasing deformation rate ($n<1$) are called <u>pseudoplastic</u> (or shear thinning) fluid.

eg :- polymer solution
Colloidal suspension.
paper pulp in water

⇒ If the apparent viscosity increases with increasing deformation rate ($n>1$) the fluid is termed <u>dilatant</u> (or shear thicking)

eg : - suspension of starch and of sand

⇒ A fluid that behaves as solid until minimum yield stress (τ_y) is exceeded and subsequently exhibits a linear relation between stress and rate of deformation is referred as <u>Bingham Plastic</u> (model law)

$$\tau = \tau_y + \mu_p \frac{du}{dy}$$

eg :- clay suspensions.
drilling muds
toothpaste
sewage sludge,
mud,
clay.

Apparent viscosity ~~is~~ may be time-dependent

Thixotropic fluid shows decrease of η with time under a constant applied shear stress.
eg:- water suspension in bentonitic clay (drilling fluid)

Rheopectic fluid show an increase ~~of~~ in η with time
eg:- gypsum pastes
printer inks.

⇒ After deformation some fluid returns to their original shape when the applied stress is

1.3 Example :- ~~A fluid has a solute viscosity~~

The velocity profile ~~for~~ a flow through a round pipe is expressed as

$$u = 2U\left(1 - \frac{r^2}{r_0^2}\right)$$

where U is the average velocity
r is the radial distance from the centre line of pipe, and r_0 is the pipe radius.

Draw the dimensionless shear stress profile $\frac{\tau}{\tau_0}$ against $\frac{r}{r_0}$, when τ_0 is the shear stress. Find the value of τ_0, when fuel oil having an ~~ab~~ visosity $\mu = 4 \times 10^{-2}\ Ns/m^2$ flow with an average velocity of 4m/s in a pipe of diameter 150 mm.

Surface Tension and Capillarity

(*) Due to molecular attraction, liquid posses certain properties such as cohesion and adhesion.

(*) Cohesion means inter-molecular attraction between molecules of the same liquid.

(*) Adhesion means attraction between the molecules of the liquid and molecules of a solid surface in contact with the liquid.

(*) The property of cohesion enables a liquid to resist tensile stress, while adhesion enables it to stick to another body.

(*) Surface tension is due to cohesion between liquid particles at the surface, whereas capillarity is due to both cohesion and adhesion.

(a) Surface tension

The property of the liquid surface film to exert a tension is called the surface tension. It is denoted by σ (Greek sigma). It is the force required to maintain unit length of the film in equilibrium.

S.I. unit of surface tension is $\frac{N}{m}$

Vapour pressure

vapour pressure is the pressure at which a liquid boils and in equilibrium with it's own vapour.

All liquid possess a tendency to evaporate or vaporize (i.e) to change from the liquid to the ~~gaseous~~ gaseous state. Such vaporization occurs because of continuous escaping of the molecules through the free liquid surface. When the liquid is confined in a closed vessel, the ejected vapour molecules get accumulated in the space between the free liquid surface and the top of the vessel.

This accumulated vapour of the liquid exerts a partial pressure on the liquid surface which is known as vapour pressure of the liquid.

(*) As molecular activity increases with temperature, vapour pressure of the liquid also increases with temperature.

(*) Mercury has a very low vapour pressure and hence it is an excellent fluid to be used in barometer. On the contrary various volatile liquids like benzene etc. have high vapour pressure.

* liquid pressure > vapour pr. → the only exchange between liquid & vapour is evaporation at the interface

* liquid pressure < vapour pr. → vapour bubbles begin to appear in the liquid

→ process is called cavitation.

water is accelerated
↓ (Turbine, water pipe) junction (geometry)
due to flow phenomena

Velocity Field

→ We studied that the continuum assumption led directly to the notion of the density field.

→ Other fluid properties may be described by fields.

→ A very important property defined by a field is the velocity field, given by

$$\vec{V} = \vec{V}(x, y, z, t) \quad \text{— eq}^n \text{ ①}$$

velocity is a vector quantity, requiring a magnitude and direction for a complete description, so the velocity field (eqⁿ(1)) is a vector field.

⇒ The velocity vector $\vec{V}$, also can be written in terms of its three scalar components.

⇒ Denoting the components in the x, y and z directions by u, v and w then.

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

⇒ In general, each component, u, v and w, will be a function of x, y, z and t

⇒ $\vec{V}(x, y, z, t)$

⇒ It indicates the velocity ~~field~~ of a fluid particle that is passing through the point (x, y, z, ~~t~~) at time instant t, in Eulerian sense.

⇒ We can keep measuring the velocity at the same point or choose any other point x, y, z at the next time instant, the point (x, y, z) is not the ongoing position of an individual particle, but a point we choose to look at.

⇒ Hence x, y and z are independent variables

⇒ $\vec{V}(x, y, z, t)$ should be thought of as the velocity field of all particles, not just the velocity of an individual particle.

⇒ If properties at every point in a flow field do not change with time, the flow is term steady. Mathematically, the defination of steady flow is

$$\frac{\partial \eta}{\partial t} = 0$$

where η represents the fluid property,

Hence for steady flow

$$\frac{\partial \rho}{\partial t} = 0 \quad \text{or} \quad \rho = \rho(x, y, z)$$

and

$$\frac{\partial \vec{V}}{\partial t} = 0 \quad \text{or} \quad \vec{V} = \vec{V}(x, y, z)$$

In steady flow, any property may vary from point to point in the field, but all properties remain constant with time at every point.

Stress field

⇒ In our study of fluid mechanics we need to understand what kind of forces act on fluid particles.

⇒ Each fluid particle can experience

(i) surface forces (pressure, friction) that are generated by contact with other particles or a solid surface

(ii) body forces (such as gravity and electromagnetic) that are experiences throughout the particle.

The gravitational body force acting on an element of volume, $d\forall$ is given by $\rho \vec{g} d\forall$

where ρ is the density (mass per unit volume)

$\vec{g}$ is the local gravitational acceleration

∴ The gravitational body force per unit volume is $\rho \vec{g}$

∴ the gravitational body force per unit mass is $\vec{g}$.

⇒ Surface forces on a fluid particle lead to stresses.

⇒ The concept of stress is useful for describing how forces acting on the boundaries of a medium (fluid or solid) are transmitted throughout the medium.

⇒ You have probably seen stresses discussed in solid mechanics.

⇒ For example :- when you stand on a diving board, stresses are generated within the board.

⇒ On the other hand, when a body moves through a fluid, stresses are developed within the fluid.

⇒ The difference between a fluid and solid is, as we have seen, that stresses in a fluid are mostly generated by motion rather than by deflection.

⇒ Imagine the surface of a fluid particle in contact with other fluid particles, and consider the contact force being generated between the particles.

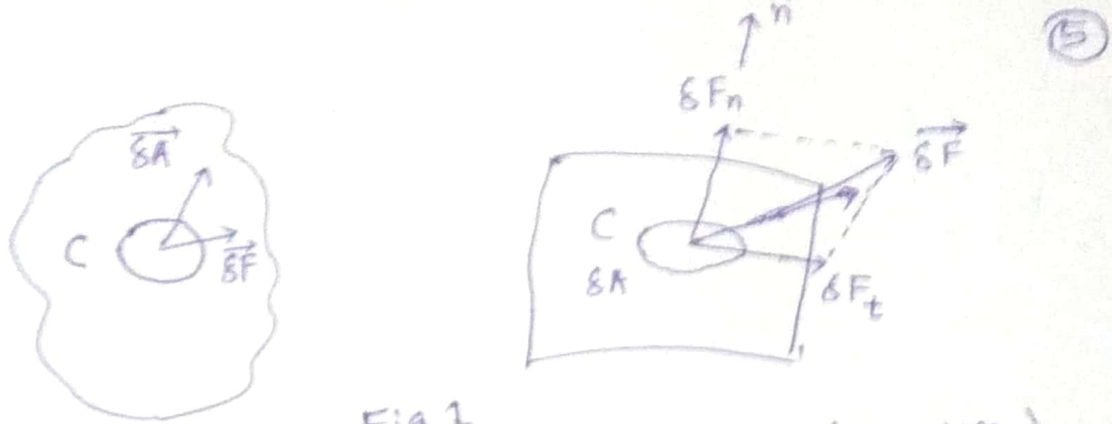
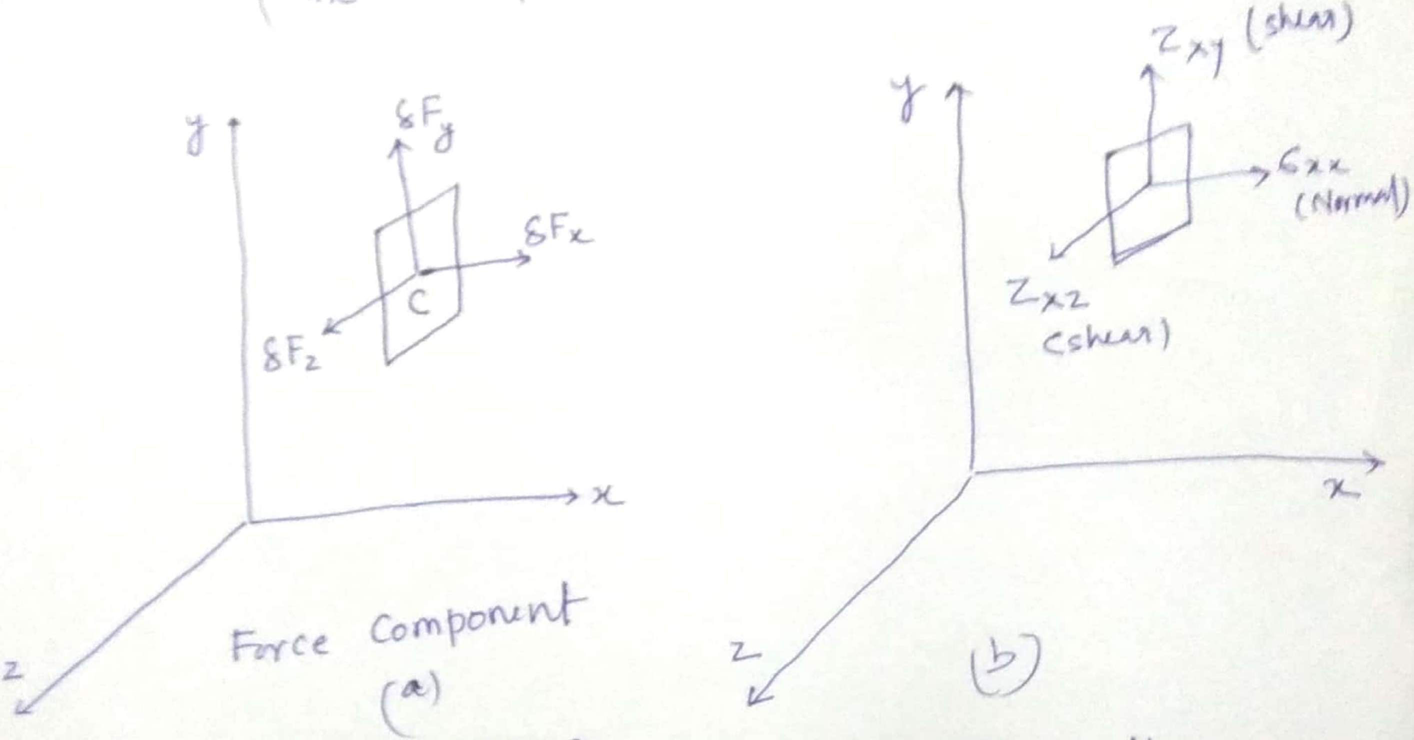


Fig 1
(The concept of stress in a continuum)



Force Component
(a)

(b)

Fig 2
(Force and stress components on the element of area δA_x)

Consider a portion, $\vec{\delta A}$ of the surface at some point c. The orientation of $\vec{\delta A}$ is given by the unit vector, $\vec{n}$, shown in Fig 1.
The vector $\hat{n}$ is the outwardly drawn unit normal with respect to the particle.

The force, $\vec{\delta F}$ acting on $\vec{\delta A}$ may be resolved into two components, one normal to and the other tangent to the area.

A normal stress σ_n and shear stress τ_n are defined as

$$\sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n} \quad \text{— (1)}$$

and $$\tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n} \quad \text{— (2)}$$

⇒ Subscript n on the stress is included as a reminder that the stresses are associated with the surface $\vec{\delta A}$ through C, having a outward normal in the $\hat{n}$ direction.

⇒ The fluid is actually a continuum, so we could have imagined breaking it up any number of different ways into fluid particle around point C, and therefore obtained any number of different stresses at point C.

⇒ In dealing with vector quantities such as force, we usually consider component in an orthogonal coordinate system. In rectangular co-ordinates we might consider the stresses acting on a planes whose outwardly drawn normals are in the x, y or z direction.

In fig 2

we consider the stresses on the element δA_x, whose outwardly drawn normal is in the x-direction. ~~The force~~ $\vec{\delta F}$

⇒ The force $\vec{\delta F}$, has been resolved into components along each of the coordinate directions.

⇒ Dividing the magnitude of each force component by the area, δA_x, and taking the limit as δA_x approaches zero.

We define the three stress components shown in fig 2(b)

$$\sigma_{xx} = \lim_{\delta A_x \to 0} \frac{\delta F_x}{\delta A_x}$$

$$\tau_{xy} = \lim_{\delta A_x \to 0} \frac{\delta F_y}{\delta A_x} \quad \text{—— (3)}$$

$$\tau_{xz} = \lim_{\delta A_x \to 0} \frac{\delta F_z}{\delta A_x}$$

We have used a double subscript notation to label the stresses.

⇒ The first subscript (in this case x) indicates the plane on which stress acts (in this case, a surface perpendicular to the x axis)

⇒ the second subscripts indicates the direction in which ~~the stress~~ acts.

⇒ consideration of an area element δA_y would ~~similarly~~ lead to the defination of stresses $\sigma_{yy}, \tau_{yx}, \tau_{yz}$

⇒ use of area element δA_z would similarly lead to the defination of $\sigma_{zz}, \tau_{zx}, \tau_{zy}$.

⇒ Although we just looked at three orthogonal planes, an infinite number of planes can be passed through point c, resulting in an infinite number of planes can be passed through point c, resulting in an infinite number of stresses associated with planes through that points

⇒ Fortunately, the state of stress at a point can be described completly by specifying the stresses acting on any three mutually perpendicular planes through the point.

⇒ The stress at a point is specified by nine components.

$$\begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

σ → normal stress.

τ → shear stress.

$\tau_{yx} = 3.5\ N/m^2$ represents a shear stress on a positive y plane in the positive x direction. or shear stress on a negative y plane in the negative x direction.

Fluid Statics

→ fluid statics often called hydrostatics (even though not restricted to water)

→ pressure generated within a static fluid is a important phenomenon in many practical situations.

→ Using the principle of hydrostatics, we can compute forces on

→ submerged objects

→ develop instruments for measuring pressure

→ determine the forces developed by hydraulic systems in applications such as industrial presses or automobile brakes.

The Basic Equation of Fluid Statics

Objective → obtain an equation for computing the pressure field in a static fluid

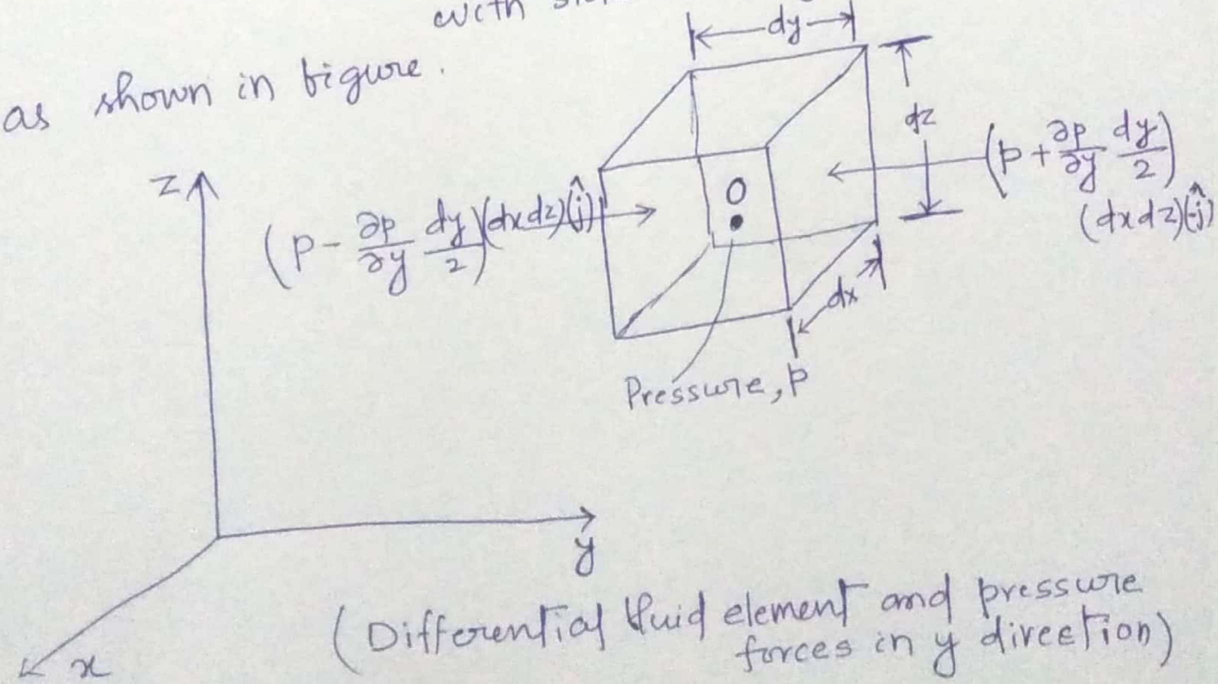
Derive → pressure increases with depth.

To derive this, we apply Newton's second law to a differential fluid element of mass

$$dm = \rho \, d\forall$$

with sides dx, dy and dz

as shown in figure.



(Differential fluid element and pressure forces in y direction)

→ The fluid element is stationary relative to the stationary rectangular co-ordinate system **shown**.

We know that

→ Two general types of forces may be applied to fluid → body forces
→ surface forces

→ The only body force that must be considered in most engineering problems is due to gravity.

→ In some situations body forces caused by electric or ~~magnitude~~ magnetic fields might be present, they will not be considered in this text.

→ For a differential fluid element, the body force is

$$d\vec{F}_B = \vec{g}\, dm = \vec{g}\, \rho\, d\forall$$

where $\vec{g}$ is the local gravity vector
ρ is the density
$d\forall$ is the volume of the element

In Cartesian co-ordinates

$$d\forall = dx\, dy\, dz$$

$$\therefore\ d\vec{F}_B = \rho \vec{g}\, dx\, dy\, dz$$

In a static fluid there are no shear stresses, so the only surface force is the pressure force.

Pressure is a scalar field, $p = p(x, y, z)$

(In general we expect the pressure to vary with position within the fluid)

→ The net pressure force that results from this variation can be found by summing the forces that act on the six faces of the fluid element.

Let the pressure p at the center, O, of the element. To determine the pressure at each of the six faces of the element, we use a Taylor series expansion of the pressure about point 'O'.

The pressure at the left face of the differential element is

$$P_L = p + \frac{\partial p}{\partial y}(y_L - y)$$

$$= p + \frac{\partial p}{\partial y}\left(-\frac{dy}{2}\right)$$

$$= p - \frac{\partial p}{\partial y}\frac{dy}{2}$$

(Terms of higher order are omitted because they will vanish in the subsequent limiting process).

The pressure on the right face of the differential element is

$$P_R = p + \frac{\partial p}{\partial y}(y_R - y)$$

$$= p + \frac{\partial p}{\partial y}\frac{dy}{2}$$

(x-z plane)

The pressure forces acting on the two y surfaces of the differential element are shown in Figure.

→ Each pressure force is a product of three factors.

* The first is the magnitude of the pressure
* (This magnitude is multiplied by the area of the face to give the magnitude of the pressure force and a unit vector is introduce to indicate direction)

→ Pressure force on each face acts against the face.

→ A positive pressure corresponds to a compressive normal stress.

→ pressure forces on the other faces of the element are obtained in the same way.

Combining all such forces gives the net surface force acting on the element.

Thus.

$$d\vec{F}_s = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right)(dy\,dz)(\hat{i}) + \left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right)(dy\,dz)(-\hat{i})$$

$$+ \left(p - \frac{\partial p}{\partial y}\frac{dy}{2}\right)(dx\,dz)(\hat{j}) + \left(p + \frac{\partial p}{\partial y}\frac{dy}{2}\right)(dx\,dz)(-\hat{j})$$

$$+ \left(p - \frac{\partial p}{\partial z}\frac{dz}{2}\right)(dx\,dy)(\hat{k}) + \left(p + \frac{\partial p}{\partial z}\frac{dz}{2}\right)(dx\,dy)(-\hat{k})$$

collecting and cancelling the terms, we obtain.

$$d\vec{F}_s = -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)dx\,dy\,dz \quad (1)$$

The term in the parenthesis is called gradient of the pressure or simply the pressure gradient and may be written grad p or ∇p. In rectangular co-ordinates.

$$\text{grad } p = \nabla p = \left(\hat{i}\frac{\partial p}{\partial x} + \hat{j}\frac{\partial p}{\partial y} + \hat{k}\frac{\partial p}{\partial z}\right)$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)p$$

The gradient can be viewed as a vector operator, taking the gradient of a scalar field gives a vector field.

Eq^n (1) can be written as

$$d\vec{F}_s = -\text{grad } p(dx\,dy\,dz)$$

$$= -\nabla p\, dx\,dy\,dz \quad \text{——} (2)$$

physically, the gradient of pressure is the negative of the surface force per unit volume due to pressure.

* <u>Note</u> * The pressure magnitude itself is not relevant in computing the net pressure force, instead what counts is the rate of change of pressure with distance, the <u>pressure gradient</u>. (important term)

We combine the bormulations for surface and body force that we have developed to obtain the total force acting on a bluid element.

Thus

$$d\vec{F} = d\vec{F}_S + d\vec{F}_B$$

$$= (-\nabla p + \rho\vec{g})\, dx\,dy\,dz$$

$$= (-\nabla p + \rho\vec{g})\, d\forall$$

or on a per unit volume basis

$$\frac{d\vec{F}}{d\forall} = -\nabla p + \rho\vec{g} \quad \text{—— (3)}$$

For a bluid particle, Newton's second law gives $\vec{F} = \vec{a}\,dm$

$$= \vec{a}\,\rho\,d\forall$$

For a static bluid, $\vec{a} = 0$.

Thus $\dfrac{d\vec{F}}{d\forall} = \rho\vec{a} = 0$

Substiting $\dfrac{d\vec{F}}{d\forall}$ in eqⁿ (3), we obtain

$$-\nabla p + \rho\vec{g} = 0 \quad \text{—— (4)}$$

· Let us review this equation briefly.

· The physical significance of each term is

$$-\nabla p \quad + \quad \rho\vec{g} = 0$$

{Net pressure force per unit volume at a point} + {body force per unit volume at a point} = 0

This is a vector equation, which means that is equivalent to three component equations that must be satisfied individually.

The component equations are

$$\left.\begin{aligned} -\frac{\partial p}{\partial x} + \rho g_x &= 0 \quad x\text{-dir}^n \\ -\frac{\partial p}{\partial y} + \rho g_y &= 0 \quad y\text{-dir}^n \\ -\frac{\partial p}{\partial z} + \rho g_z &= 0 \quad z\text{-dir}^n \end{aligned}\right\} \quad -(5)$$

Eq^n 5 describe the pressure variation in each of the three coordinate directions in a static fluid.

→ It is convenient to choose a coordinate system such that the gravity vector is aligned with one of the co-ordinate axes.

→ If the coordinate system is chosen with the z-axis directed vertically upward as shown in figure, then $g_x = 0$, $g_y = 0$ and

$g_z = -g$.

Under these conditions, the component equations become

$$\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\rho g \quad \text{—(6)}$$

Eqn (6) indicates that, under the assumption made, the pressure is independent of coordinates x and y, it depends on z alone.

∴ Thus since p is a function of a single variable, a total derivative may be used instead of partial derivative, With this simplification eqn (6) becomes

$$\boxed{\frac{dp}{dz} = -\rho g = -\gamma} \quad \text{—(7)}$$

γ → specific weight of the fluid

This eqn is the basic pressure-height relation of fluid statics.

Restriction

1. static fluid
2. Gravity is the only body force
3. z axis is vertical and upward

⇒ To determine pressure distribution in a static fluid eqⁿ (7) may be integrated and appropriate boundary conditions applied.

Hydrostatic Force On Submerged Surfaces

We determined how the pressure varies in a static fluid, we can examine the force on surface submerged in liquid.

In order to determine completely the resultant force acting on a submerged surface, we must specify:

1. The magnitude of the force
2. The direction of the force
3. The line of action of the force

We shall consider both plane and curved submerged surfaces.

Hydrostatic Force on a Plane Submerged Surface

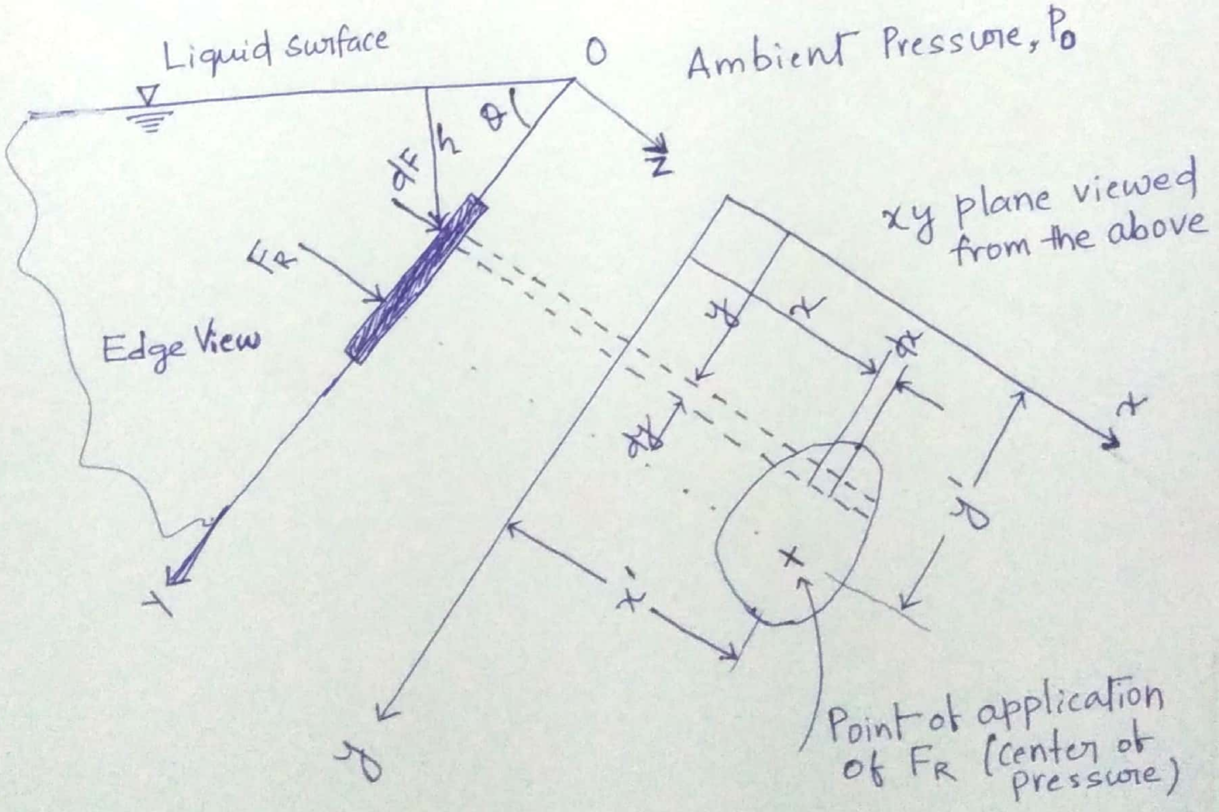


Figure: Plane Submerged surface.

→ A plane submerged surface, on whose upper face we wish to determine the resultant hydrostatic force ~~as~~ is shown in figure.

→ The co-ordinates are important: They have been chosen so that the surface lies in the xy plane, and the origin 'O' is located at the intersection of the plane surface and the free surface.

→ As well as the magnitude of the force F_R, we wish to locate the point (with co-ordinates x', y') through which it acts on the surface.

→ Since there are no shear stress on the static fluid, the hydrostatic force on any element of the surface acts normal to the surface.

→ The pressure force acting on any element $dA = dx\,dy$ of the upper surface is given by

$$dF = p\,dA$$

→ The resultant force acting on the surface is found by summing the contributions of the infinitesimal forces over the entire area.

→ Usually when we sum forces we must do so in a vertical sense. However, in this case all of the infinitesimal forces are perpendicular to the plane, and hence so ~~is~~ is the resultant force.

Its magnitude is given by.

$$F_R = \int_A p\, dA \qquad \text{— (1)}$$

In order to evaluate the integral in eqn (1), both the pressure, p, and the element of the area, dA, must be expressed in terms of the same variables

∴ The pressure p at depth h in the liquid as

$$p = P_0 + \rho g h$$

where P_0 is the pressure at the free surface (h = 0)

In addition, we have, from the system geometry, $h = y \sin\theta$

Using this expression and the expression of pressure in eqn (1)

$$F_R = \int_A p\, dA = \int_A (P_0 + \rho g h)\, dA$$

$$= \int_A (P_0 + \rho g y \sin\theta)\, dA$$

$$F_R = P_0 \int_A dA + \rho g \sin\theta \int_A y\, dA = P_0 A + \rho g \sin\theta \int_A y\, dA$$

The integral is the first moment of the surface area about the x axis, which may be written as

$$\int_A y\, dA = y_c A$$

y_c is the y co-ordinate of the centroid of the area A.

Thus $F_R = P_0 A + \rho g \sin\theta\, y_c A = (P_0 + \rho g h_c) A$

$$F_R = P_c A \qquad — (2)$$

where P_c is the absolute pressure in the liquid at the location of the centroid of area A.

⇒ Eq(2) computes the resultant force due to the liquid — including the effect of the ambient pressure P_0 — on one side of the submerged plane surface.

⇒ It does not take into account whatever pressure or force distribution may be on the other side of the surface.

⇒ However, if we have the same pressure, P_0, on this side as we do at the free surface of the liquid as shown figure below, its effect on F_R cancels out, and if we wish to obtain the net

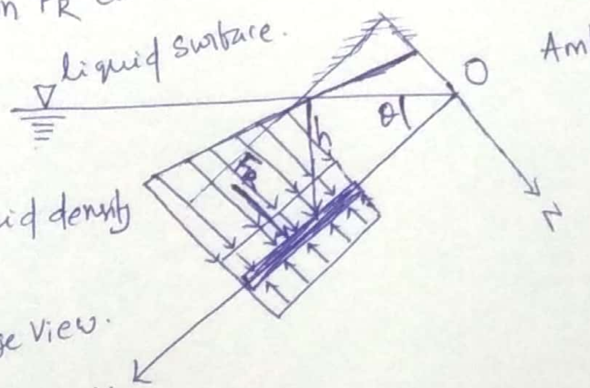


Figure :- Pressure distribution on plane submerged surface.

force on the surface we can use eqn (2) with P_c expressed as a gage rather than absolute pressure.

⇒ In computing F_R we can use either the integral of eqn(1) or the resulting eqn(2).

(It is important to note that even though the force can be computed using the pressure at the center of the plate, this is not the point through which the force acts)

Our next task is to determine (x', y'), the location of the resultant force.

Lets first obtain y' by recognizing that the moment of the resultant force about the x axis must be equal to the moment due to the distributed pressure force.

⇒ Taking the sum (i.e. integral) of the moments of the infinitesimal force dF about the x-axis we obtain

$$\boxed{y' F_R = \int_A y p \, dA} \quad \text{— (3)}$$

We can integrate by expressing p as a function of y as

$$y' F_R = \int_A y p \, dA = \int_A y (P_0 + \rho g h) \, dA$$

$$= \int_A (P_0 y + \rho g y^2 \sin\theta) \, dA$$

$$= P_0 \underbrace{\int_A y \, dA}_{y_c A} + \rho g \sin\theta \underbrace{\int_A y^2 \, dA}_{\text{second momentum area about } x \text{ axis } I_{xx}}$$

We can use the parallel axis theorem.

$$I_{xx} = I_{\hat{x}\hat{x}} + A y_c^2$$

↑ centroidal axis.

(To relplace I_{xx} with the standard second moment of area, about the centroidal ~~axis~~ $\hat{x}$ axis.)

Using all these, we find

$$y' F_R = P_o y_c A + \rho g \sin\theta \left(I_{\hat{x}\hat{x}} + A y_c^2 \right)$$

$$= y_c \left(P_o + \rho g y_c \sin\theta \right) A + \rho g \sin\theta \, I_{\hat{x}\hat{x}}$$

$$= y_c \left(P_o + \rho g h_c \right) A + \rho g \sin\theta \, I_{\hat{x}\hat{x}}$$

$$= y_c F_R + \rho g \sin\theta \, I_{\hat{x}\hat{x}}$$

Finally, we obtain for y'

$$y' = y_c + \frac{\rho g \sin\theta \, I_{\hat{x}\hat{x}}}{F_R} \quad \text{—(4)}$$

Eq^n (4) is convenient for computing the location y' of the force on the submerged side of the surface when we include the ambient pressure P_o.

⇒ If we have the same ambient pressure acting on the other side of the surface we can use eq^n (3) with P_o neglected to compute the net force.

$$F_R = P_{c\,gage} A = \rho g h_c A = \rho g y_c \sin\theta A$$

Eq^n(4) becomes for this case.

$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c} \quad \text{—(5)}$$

Eqn (3) is the integral eqn for computing the location y' of the resultant force.

Eqn (4) is the useful algebric form for computing y' when we are interested in the resultant force on the submerged side of the surface

Eqn (5) is for computing y' when we are interested in the net force for the case when the same P_0 acts at the free surface and on the otherside of the submerged surface.

For problems that have a pressure on the other side is that is not P_0, we can analyze each side of the surface seperately or reduce the two pressure distributions to one net pressure distribution, in effect creating a system to be solved using eqn (3) with P_c expressed as gage pressure.

Note that in any event $y' > y_c$, the location of the force is always below the level of the plate centroid. This make sense — as in fig(2), the pressure will always be larger on the lower regions, moving the resultant force down the plate

A similar analysis can be done to compute the x', the x-location of the force on te plate

Taking the sum of te moments of te infinitesmal force dF about te y axis we obtain.

$$\boxed{x' F_R = \int_A x P \, dA} \quad \text{—} \; (6)$$

we can express p as a function of y as before

$$x'R = \int_A x p \, dA$$

$$= \int_A x (P_0 + \rho g h) \, dA$$

$$= \int_A (P_0 x + \rho g x y \sin\theta) \, dA$$

$$= P_0 \underbrace{\int_A x \, dA}_{x_c A} + \rho g \sin\theta \underbrace{\int_A x y \, dA}_{I_{xy}}$$

$x_c \rightarrow$ in the distance of the centroid from y axis

Using the parallel axis theorem

$$I_{xy} = I_{\hat{x}\hat{y}} + A x_c y_c$$

we find

$$x' F_R = P_0 x_c A + \rho g \sin\theta \left(I_{\hat{x}\hat{y}} + A x_c y_c \right)$$

$$= x_c \left(P_0 + \rho g y_c \sin\theta \right) A + \rho g \sin\theta \, I_{\hat{x}\hat{y}}$$

$$= x_c \left(P_0 + \rho g h_c \right) A + \rho g \sin\theta \, I_{\hat{x}\hat{y}}$$

$$= x_c F_R + \rho g \sin\theta \, I_{\hat{x}\hat{y}}$$

Finally, we obtain

$$\boxed{x' = x_c + \frac{\rho g \sin\theta \, I_{\hat{x}\hat{y}}}{F_R}} \quad -(7)$$

If the ambient pressure also acting on the other side of the surface we can again use eqn (3) with P_0 neglected to compute the net force and eqn (7) becomes for this case

$$\boxed{x' = x_c + \frac{I_{\hat{x}\hat{y}}}{A y_c}} \quad — (8)$$

Direction of force will always be perpendicular to the plane

Hydrostatic Force on a Curved Submerged Surface

→ For curved surfaces, we will once again derive expressions for the resultant force by integrating the pressure distribution over the surface.

→ However, unlike for the plane surface, we have a more complicated problem — the pressure force is normal to the surface at each point, but now the infinitesimal area elements point in varying directions, because of the surface curvature.

→ This means that instead of integrating over an element dA we need to integrate over the vector element $d\vec{A}$

→ This will initially lead to a more complicated analysis, but we will see that a simple solution technique will be developed.

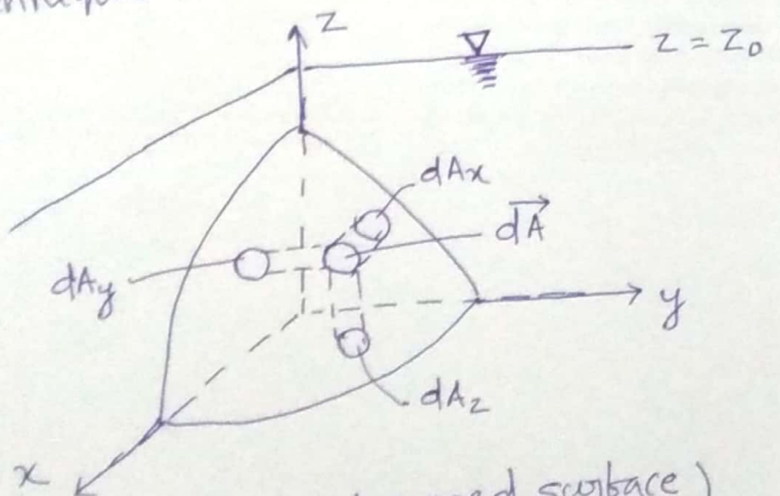


Fig 1. (Curved submerged surface)

→ Consider the curved surface shown in figure 1.

The pressure force acting on the element of area, $\vec{dA}$, is given by

$$\vec{dF} = -p\,\vec{dA}$$

where the -ve sign indicates that the force acts on the area, in a direction opposite to the area normal.

The resultant force is given by

$$\vec{F}_R = -\int_A p\,\vec{dA} \quad \text{——— (1)}$$

We can write

$$\vec{F}_R = \hat{i}\,F_{Rx} + \hat{j}\,F_{Ry} + \hat{k}\,F_{Rz}$$

where F_{Rx}, F_{Ry} and F_{Rz} are the components of $\vec{F}_R$ in the positive x, y and z directions, repectively

→ To evaluate the component of the force in a given direction, we take the dot product of the force with the unit vector in the given direction.

→ For example, taking the dot product of each side of equation (1) with the unit vector $\hat{i}$ gives.

$$F_{Rx} = \vec{F}_R \cdot \hat{i} = \int d\vec{F} \cdot \hat{i} = -\int_A p\,\vec{dA} \cdot \hat{i}$$

$$= -\int_{A_x} p\,dA_x$$

$$F_{Rx} = \vec{F}_R \cdot \hat{i} = \int d\vec{F} \cdot \hat{i} = -\int_A p\, d\vec{A} \cdot \hat{i} = -\int_{A_x} p\, dA_x$$

where dA_x is the projection of $d\vec{A}$ on a plane perpendicular to x-axis. (Refer figure 1)

and -ve sign indicates that the x component of the resultant force is in the negative x direction.

⇒ Since in any problem, the direction of the force component can be determined by inspection, the use of vectors is not necessary.

⇒ In general, the magnitude of the component of the resultant force in the l direction is given by

$$F_{Rl} = \int_{A_l} p\, dA_l \qquad (2)$$

where dA_l is the projection of the area element dA on a plane perpendicular to 'l' direction.

⇒ The line of action of each component of the resultant force is found by recognizing that the moment of the resultant force component about a given axis must be equal to the moment of the corresponding distributed force component about the same axis.

Eqⁿ (2) can be used for the horizontal forces F_{Rx} and F_{Ry}.

→ We have the interesting result that the horizontal force and its location are the same as for an imaginary vertical plane surface of the same projected area. This is illustrated in fig 2. where we have called the horizontal force F_H.

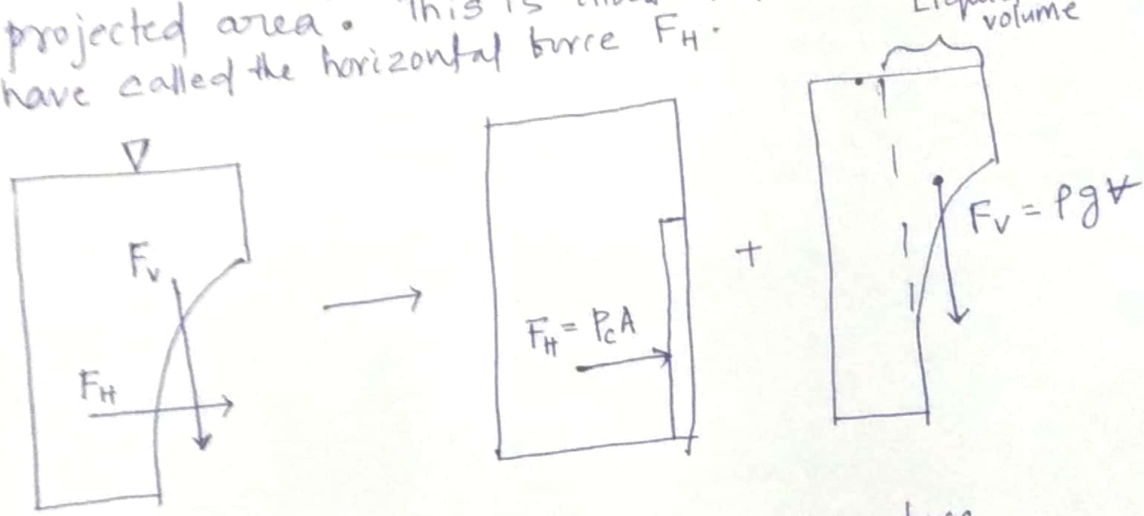


Fig2:- Forces on curved submerged surface.

Fig (2) also illustrates how we can compute the vertical component of force:

→ With atmospheric pressure at the free surface and on the other side of the curved surface the net vertical force will be equal to the weight of the fluid directly above the surface.

→ This can be seen by applying eqⁿ(2) to determine the magnitude of the vertical component of the resultant force, obtaining

$$F_{Rz} = F_v = \int p \, dA_z$$

Since $P = \rho g h$

$$F_v = \int \rho g h \, dA_z$$

$$= \int \rho g \, d\forall$$

where $\rho g h \, dA_z = \rho g \, d\forall$ is the weight of a differential cylinder of the liquid above the element of surface area, dA_z, extending a distance h from the curved surface to the free surface.

The vertical component of the resultant force is obtained by integrating over the entire submerged surface. Thus

$$F_v = \int_{A_z} \rho g h \, dA_z = \int_{\forall} \rho g \, d\forall = \rho g \forall$$

In summary, for a curved surface we can use two simple formulas for computing the horizontal and vertical force components due to the fluid only (no ambient pressure)

$$\left. \begin{aligned} F_H &= P_c A \\ F_v &= \rho g \forall \end{aligned} \right\} \quad \text{— (3)}$$

where P_c — Pressure at the center

A — Area of a vertical plane surface of the same projected area

$\forall$ → volume of the fluid above the curved surface.

⇒ It can be shown that the line of action of the vertical force component passes through the center of gravity of the volume of liquid directly above the curved surface

⇒ We have shown that the resultant hydrostatic force on a curved submerged surface is specified in terms of its components.

Vortex flow :-

Vortex flow is defined as the flow of fluid along a ~~closed~~ curved path / or flow of rotating mass of fluid is known as vortex flow.

The vortex flow is of two types
1. Forced vortex flow
2. Free vortex flow

Forced vortex flow : Forced vortex flow is defined as that type of vortex flow in which some external torque is required to rotate the fluid mass.

The fluid mass in this type of flow, rotate at a constant angular velocity, ω. The tangential velocity of any fluid particle is given by

$$v = \omega \times r.$$

$r \rightarrow$ Radius of fluid particle from the axis of rotation.

Example of forced vortex flow :-

1. Vertical cylinder containing liquid which is rotated about its central axis with a constant angular velocity.
2. Flow of liquid inside the impeller of a centrifugal pump.
3. Flow of water through the runner of turbine.

Free Vortex flow :-

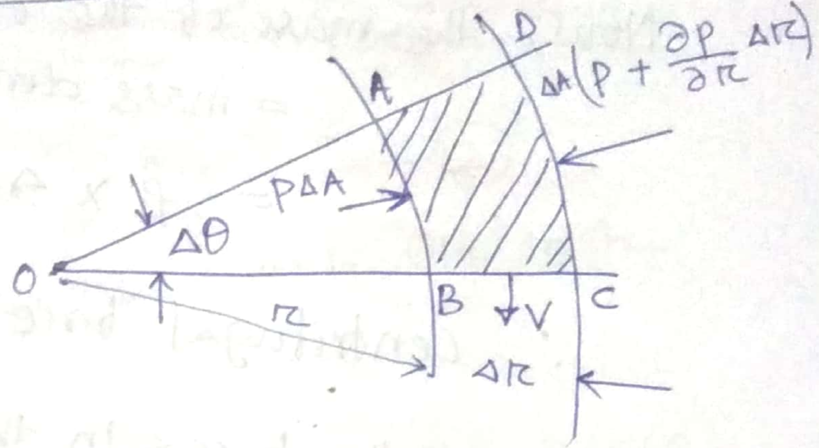
when no external torque is required to rotate the fluid mass, the type of fluid flow is called free vortex flow.

(Thus the liquid in case of free vortex is rotating due to rotation which is imparted to the fluid previously.)

Example of free vortex flow

1. Flow of liquid through a hole provided at the bottom of a container
2. Flow of liquid around a circular bend in pipe.
3. A whirlpool in a river.
4. Flow of fluid in a centrifugal pump casing.

Equation of motion for vortex flow



Consider a fluid element ABCD rotating at a uniform velocity in a horizontal plane about an axis perpendicular to the plane of paper and passing through O.

Let r = Radius of the element from O

$\Delta\theta$ = Angle subtended by the element at O

Δr = Radial thickness of the element

ΔA = Area of cross-section of the element.

The forces acting on the element are

(i) pressure force, $p\Delta A$ on the face AB

(ii) pressure force, $\left(p + \frac{\partial p}{\partial r}\Delta r\right)\Delta A$ on face CD

(iii) Centrifugal force, $\frac{mv^2}{r}$ acting in the direction away from the centre, O

Now, the mass of the element

= mass density x volume

$= \rho \times \Delta A \times \Delta r$

$\therefore$ centrifugal force $= \rho \, \Delta A \, \Delta r \, \frac{V^2}{r}$.

Equating the forces in the radial direction, we get

$$\left(p + \frac{\partial p}{\partial r} \Delta r\right) \Delta A - p \Delta A = \rho \, \Delta A \, \Delta r \, \frac{V^2}{r}$$

$$\Rightarrow \frac{\partial p}{\partial r} \Delta r \, \Delta A = \rho \, \Delta A \, \Delta r \, \frac{V^2}{r}$$

Cancelling $\Delta r \, \Delta A$ to both side, we get

$$\frac{\partial p}{\partial r} = \rho \frac{V^2}{r} \quad \text{——— (1)}$$

eqn (1) gives the pressure variation along the radial direction for a forced or free vortex flow in a horizontal plane.

$\frac{\partial p}{\partial r} \rightarrow$ pressure gradient along the radial direction.

$p \rightarrow$ increases with increse in r

The pressure variation in the vertical plane is given by hydrostatic law (i.e)

$$\frac{\partial p}{\partial z} = -\rho g \quad \text{--- (2)}$$

In eqn (2), z is measured vertically in the upward direction.

∴ p is a function of r and z, hence total derivative of p is

$$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} dz$$

substituting the value of $\frac{\partial p}{\partial r}$ and $\frac{\partial p}{\partial z}$ from eqn (1) & (2), we get

$$dp = \rho \frac{v^2}{r} dr - \rho g dz \quad \text{--- (3)}$$

Eqn (3) gives the variation of pressure of a rotating fluid in any plane.

Equation of forced vortex flow

For a forced vortex flow

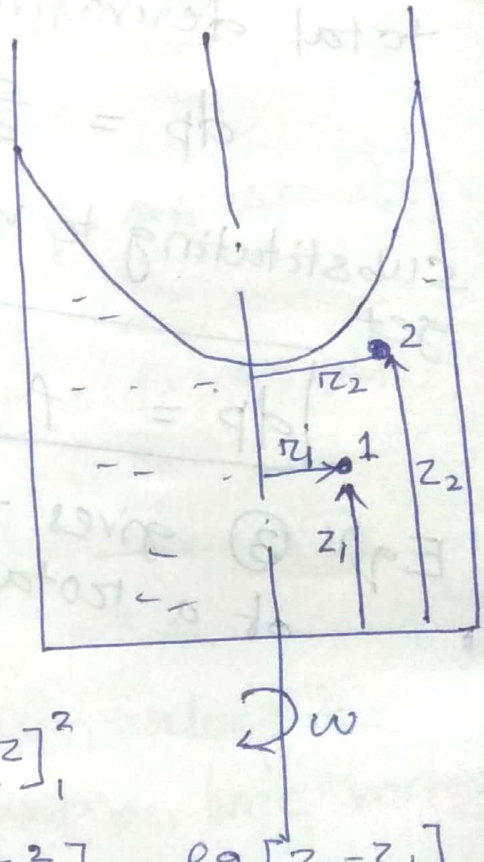
we have $V = \omega \times r$.

$\omega \rightarrow$ Angular velocity = constant.

Substituting the value of v in eqn (3), we get

$$dp = \rho \times \frac{\omega^2 r^2}{r} dr - \rho g dz \quad \text{---(4)}$$

Consider two points 1 and 2 in the fluid having forced vortex flow.



Integrating eqn (4)

$$\int_1^2 dp = \int_1^2 \rho \omega^2 r\, dr - \int_1^2 \rho g dz$$

$$= \left[\rho \omega^2 \frac{r^2}{2}\right]_1^2 - \rho g [z]_1^2$$

$$\Rightarrow P_2 - P_1 = \frac{\rho}{2}\left[\omega^2 r_2^2 - \omega^2 r_1^2\right] - \rho g [z_2 - z_1]$$

$$= \frac{\rho}{2}\left[v_2^2 - v_1^2\right] - \rho g (z_2 - z_1) \quad \text{---(5)}$$

$\because$ $v_2 = \omega r_2$

$v_1 = \omega r_1$

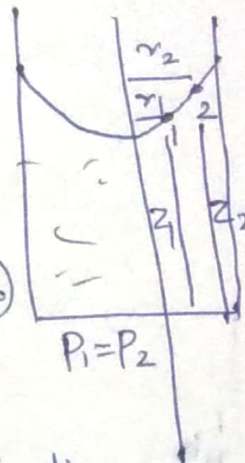
If points 1 and 2 on the free surface of the liquid, then $P_1 = P_2$

Hence eqn (5) becomes.

$$0 = \frac{\rho}{2}\left[V_2^2 - V_1^2\right] - \rho g\left[z_2 - z_1\right]$$

$$\rho g(z_2 - z_1) = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$\Rightarrow \boxed{z_2 - z_1 = \frac{1}{2g}(V_2^2 - V_1^2)} \quad (6)$$



If point one lies on the axis of rotation.

$$V_1 = \omega r_1 = \omega \times 0 = 0$$

$$\therefore z_2 - z_1 = \frac{1}{2g} V_2^2$$

$$= \frac{V_2^2}{2g}.$$

If $z_2 - z_1 = Z$

then $$Z = \frac{V_2^2}{2g} = \frac{\omega^2 \times r_2^2}{2g}. \quad (7)$$

$\therefore$ Z varies with the square of r.

Hence eqn (7) is an eqn of parabola.

$\therefore$ Free surface of liquid is paraboloid

Module-II (Lec 1)

Basic Equations in Integral Form for a Control Volume :

* Differential Analysis to fluid Motion

→ study fluid in motion

→ examine a flowing fluid

Two Methods

(1) → Study the motion of an individual fluid particle or group of particles as they move through space.

→ this is the system approach (Lagrangian approach)

→ its advantage is that the physical laws apply to matter and hence directly to the system.

e.g:- Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt}$$

where $\vec{F}$ → force

$\frac{d\vec{p}}{dt}$ → rate of momentum change of the fluid

→ its disadvantage is that the math associated with this approach can become somewhat complicated, leading to ~~the~~ a set of partial differential equations.

→ The system approach is needed if we are interested in studying the trajectory of particles over time.

(e.g. in pollution studies)

(2) → Study a region in space as fluid flows through it

→ this is the **Control volume approach.** (Eulerian approach)

→ this is very often the method of choice, because it has widespread practical application (because of its continuously deformable nature)

(e.g.) in aerodynamics we are usually interested in the lift and drag on a wing (which we select as part of the control volume) rather than what happens to individual fluid particles.

→ its disadvantage is that the physical laws apply to matter and not directly to the region of space, so we have to perform some math (mathematical transformation) to convert physical laws from system formulation to the control volume formulation.

(3)

integral → study finite region in space

differential → study motion of a particle → differential control volume

→ This is an important distinction

Integral → It indicates that we will study a finite region ~~in~~ ~~space~~ space

Differential → study the motion of a particle (an infinitesimal) differential control volume to derive famous Bernoulli eqⁿ)

~~Law~~→ (although we study [chapter 4]

Agenda of this chapter (integral form) is to review the physical laws as they apply to a system, develop some math to convert form of a system to control volume description and obtain formulas for the physical laws for control volume analysis.

④ Basic Laws for a System

The basic law we will apply are

- conservation of mass
- Newton's second law
- the angular-momentum principle
- First and second laws of thermodynamics

For converting these system equations to equivalent control volume formulas, it turns out we want to express each of the laws as a rate equation.

(1) Conservation of Mass

For a system (by defination a fixed amount of matter, M, we have chosen) we have the simple result that

$$M = \text{constant}$$

However, as discussed above, we wish to express each physical law as a rate equation, so we write

$$\left.\frac{dM}{dt}\right)_{system} = 0$$

where $M_{system} = \int_{M(system)} dm = \int_{\forall(system)} \rho \, d\forall$

(2) Newton's Second Law

For a system moving relative to an inertial reference frame,

Newton's second law states that the sum of all external forces acting on the system is equal to the time rate of change of linear momentum of the system,

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right)_{system}$$

where the linear momentum of the system is given by

$$\vec{P}_{system} = \int_{M(system)} \vec{V}\,dm = \int_{\forall(system)} \vec{V}\rho\,d\forall$$

(velocity)

3) The Angular-Momentum Principle

The angular-momentum principle for a system states that the rate of change of angular momentum is equal to the sum of all torques acting on the system,

$$\vec{T} = \left.\frac{d\vec{H}}{dt}\right)_{system}$$

where the angular momentum of the system is given by

$$\vec{H}_{system} = \int_{M(system)} \vec{r}\times\vec{V}\,dm = \int_{\forall(system)} \vec{r}\times\vec{V}\rho\,d\forall$$

Torque can be produced by surface and body forces (here gravity) and also by shafts that cross the system boundary

$$\vec{T} = \vec{r}\times\vec{F}_s + \int_{M(system)} \vec{r}\times\vec{g}\,dm + \vec{T}_{shaft}$$

4) The First Law of Thermodynamics

The first law of thermodynamics is a statement of conservation of energy for a system,

$$\delta Q - \delta W = dE$$

The equation can be written in rate form as

$$\dot{Q} - \dot{W} = \left.\frac{dE}{dt}\right)_{system}$$

Where the total energy of the system is given by

$$E_{system} = \int_{M(system)} e\,dm = \int_{\forall(system)} e\rho\,d\forall$$

and

$$e = u + \frac{V^2}{2} + gz$$

$u \rightarrow$ specific internal energy
$V \rightarrow$ speed
$z \rightarrow$ height (from datum) of a particle having mass (dm)

$\dot{Q} \rightarrow$ the rate of heat transfer is positive when heat is added to the system from the surroundings

$\dot{W} \rightarrow$ the rate of work is positive when work is done by the system on its surroundings

5) The Second Law of Thermodynamics

If an amount of heat, δQ, is transferred to a system at temperature T, the second law of thermodynamics states that the change in entropy, ds, of the system satisfies

$$ds \geq \frac{dQ}{T}$$

On a rate basis we can write

$$\left.\frac{ds}{dt}\right)_{system} \geq \frac{1}{T}\dot{Q}$$

where the total entropy of the system is given by

$$S_{system} = \int_{M(system)} s\,dm = \int_{\forall(system)} s\,\rho\,d\forall$$

(1)

Relation of System Derivatives to the Control Volume Formulation :—

⇒ We now have five basic laws expressed as system rate equations.

⇒ Our task in this section is to develop a general expression for converting a system rate equation into an equivalent control volume equation

⇒ Instead of converting the equations for rates of change of $M, \vec{P}, \vec{H}, E$ and S one by one, we let all of them be represented by the symbol N.

⇒ Hence N, represents the amount of mass or momentum, or angular momentum or energy or entropy of the system.

⇒ Corresponding to this extensive property, we will also need the intensive (i.e. per unit mass) property η.

Thus

$$N_{system} = \int_{M(system)} \eta \, dm = \int_{\forall(system)} \eta \rho \, d\forall \quad —(1)$$

Compairing this eq^n with the previous eq^n(1-5) we, see that if

$N = M$, then $\eta = 1$

$N = \vec{P}$, then $\eta = \vec{V}$

$N = \vec{H}$, then $\eta = \vec{r} \times \vec{V}$

$N = E$, the $\eta = e$

$N = S$, the $\eta = s$

(2)

(2)

How can we derive a control volume description from a system description of a fluid flow?

⇒ Before specifically answering this question, we can describe the derivation in general terms.

⇒ We imagine selecting an arbitrary piece of the flowing fluid at some time t_0, as shown in Figure 1(a) (we could imagine dyeing this piece of fluid, say, blue.)

⇒ This initial shape of the fluid system is chosen as our control volume, which is fixed in space relative to co-ordinates xyz.

⇒ After an infinitesimal time Δt the system will have moved (probably changing shape as it does so) to a new location, as shown in Figure 1(b).

⇒ The laws we discussed above apply to this piece of fluid - for example, its mass will be constant (fig 1(a).)

⇒ By examining the geometry of the system/control volume pair at $t = t_0$ and $t = t_0 + \Delta t$, we will be able to obtain control volume formulations of the basic laws.

③

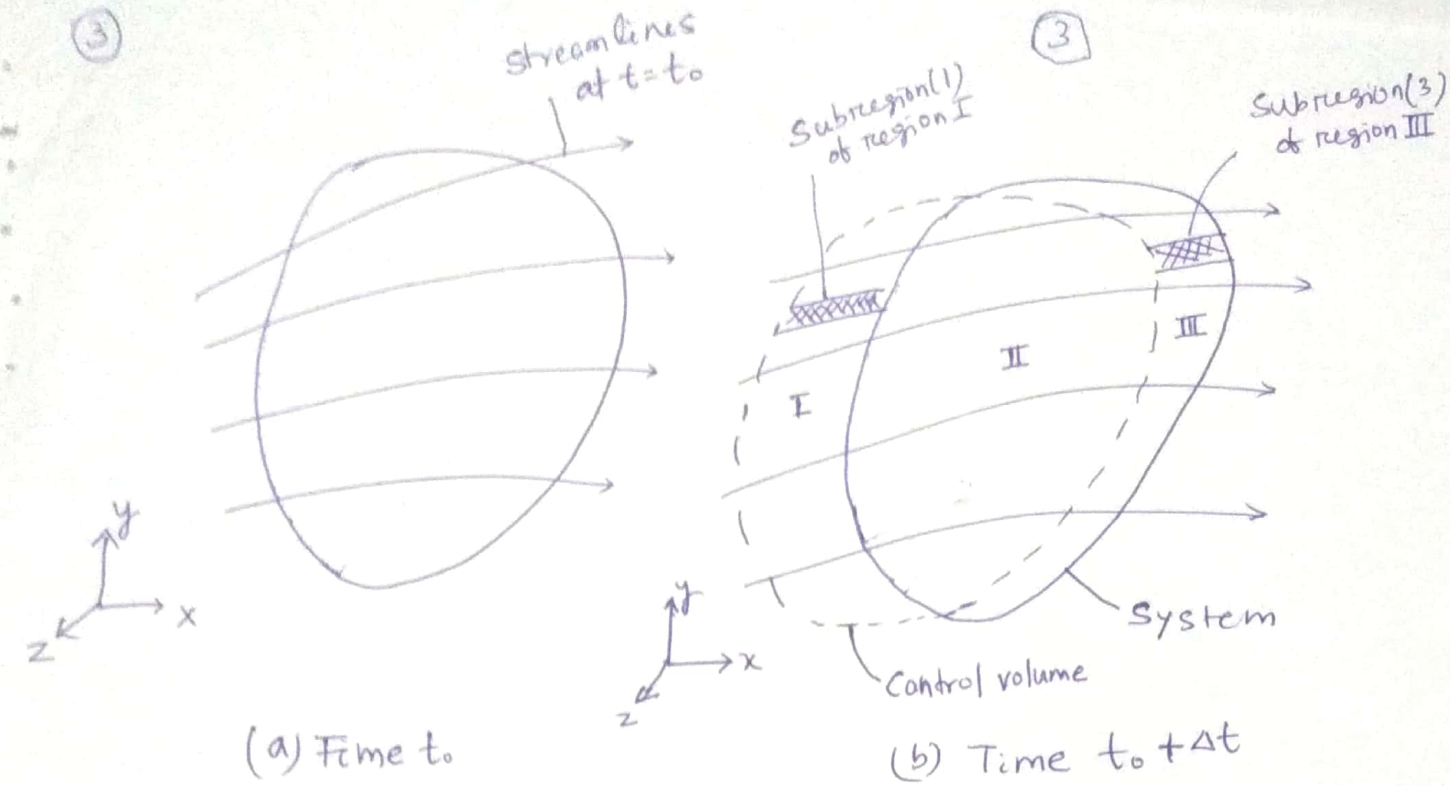


Fig 1

Derivation

⇒ In Fig 1(a&b) we see that the system, which was entirely within the control volume at time t_0, is partially out of the control volume at time $t_0+\Delta t$.

⇒ Infact, three regions can be identified. These are: regions I and II, which together make up the control volume, and region III, which, with region II, is the location of the system at time $t_0+\Delta t$.

⇒ Our objective is to relate the rate of change of any arbitrary extensive property, N, of the system to quantities associated with the control volume.

From the defination of a derivative, the rate of change of Nsystem is given by

$$\left.\frac{dN}{dt}\right)_{system} = \lim_{\Delta t \to 0} \frac{N_s)_{t_0+\Delta t} - N_s)_{t_0}}{\Delta t} \quad -(2)$$

For convenience, subscript s has been used to denote the system in the defination of a derivative in eqn (2)

From the geometry of Fig 1 ~~(Fig 2 b)~~

$$N_s)_{t_0+\Delta t} = (N_{II} + N_{III})_{t_0+\Delta t}$$

$$= (N_{cv} - N_I + N_{III})_{t_0+\Delta t}$$

and $N_s)_{t_0} = N_{cv})_{t_0}$

Substituting into the defination of the system derivative eqn (2), we obtain

$$\left.\frac{dN}{dt}\right)_s = \lim_{\Delta t \to 0} \frac{N_{cv} - N_I + N_{III})_{t_0+\Delta t} - N_{cv})_{t_0}}{\Delta t}$$

Since the limit of a sum is equal to sum of the limits, we can write

$$\left.\frac{dN}{dt}\right)_s = \underbrace{\lim_{\Delta t \to 0} \frac{N_{cv})_{t_0+\Delta t} - N_{cv})_{t_0}}{\Delta t}}_{(I)} + \underbrace{\lim_{\Delta t \to 0} \frac{N_{III})_{t_0+\Delta t}}{\Delta t}}_{(II)} - \underbrace{\lim_{\Delta t \to 0} \frac{N_I)_{t_0+\Delta t}}{\Delta t}}_{(III)} \quad -(3)$$

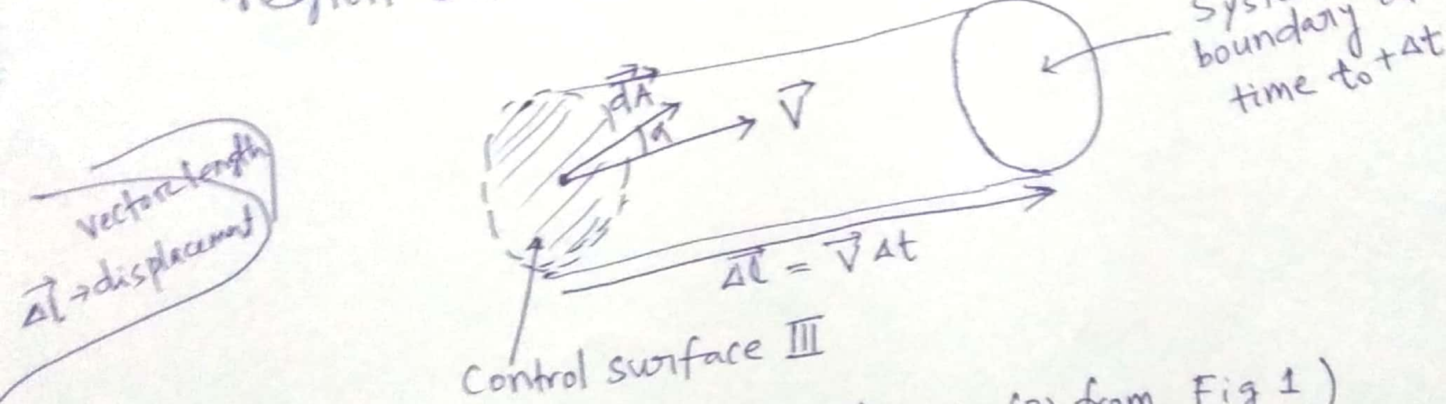
Our task now is to evaluate each of the three terms in eqn (3)

Term (I) in eq^n (3) simplifies to

$$\lim_{\Delta t \to 0} \frac{N_{cv})_{t_0+\Delta t} - N_{cv})_{t_0}}{\Delta t} = \frac{\partial N_{cv}}{\partial t}$$

$$= \frac{\partial}{\partial t} \int_{CV} \eta \rho \, d\forall \quad (4a)$$

To evaluate term (II) in eq^n (3), we first develop an expression for $N_{III})_{t_0+\Delta t}$ by looking at the enlarge view of a typical subregion (subregion 3) of region III shown in figure 2.



(Enlarge view of subregion (3) from Fig 1)

The vector area element $\vec{dA}$ of the control surface has magnitude dA, and its direction is the outward normal of the area element. In general, the velocity vector $\vec{V}$ will be at some angle α with respect to $\vec{dA}$.

⑥

For this subregion, we have ⑥

$$dN_{III})_{t_0+\Delta t} = \eta \rho \, d\forall)_{t_0+\Delta t}$$

We need to obtain an expression for the volume $d\forall$ of this cylindrical element.

The vector length of the cylinder is given by

$$\vec{\Delta l} = \vec{V} \Delta t$$

The volume of the prismatic cylinder, whose area $\vec{dA}$ is at an angle α to its length $\vec{\Delta l}$, is given by

$$d\forall = \Delta l \, dA \cos\alpha$$

$$= \vec{\Delta l} \cdot \vec{dA}$$

$$= \vec{V} \cdot \vec{dA} \, \Delta t$$

$\vec{\Delta l} = \vec{V} \Delta t$

Hence for subregion ③ of region III, we can write

$$dN_{III})_{t_0+\Delta t} = \eta \rho \underbrace{\vec{V} \cdot \vec{dA} \, \Delta t}_{d\forall}$$

Then for the entire region III we can integrate and for term (II) in eqⁿ ③ we obtain.

$$\lim_{\Delta t \to 0} \frac{N_{III})_{t_0+\Delta t}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\int_{CS_{III}} dN_{III})_{t_0+\Delta t}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\int_{CS_{III}} \eta \rho \vec{V} \cdot d\vec{A} \, \Delta t}{\Delta t} \quad (7)$$

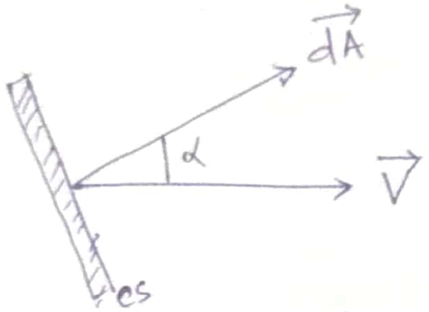
$$= \int_{CS_{III}} \eta \rho \vec{V} \cdot d\vec{A} \quad \text{— (4b)}$$

We can perform a similar analysis for subregion (1) of region I, an obtain for term (III)(3) in eqn (3)

$$\lim_{\Delta t \to 0} \frac{N_I)_{t_0 + \Delta t}}{\Delta t} = - \int_{CS_I} \eta \rho \vec{V} \cdot d\vec{A} \quad \text{— (4c)}$$

For subregion(1) of region I, the velocity vector acts into the control volume, but the area normal always (by convention) points outward (angle $\alpha > \pi/2$), so the scalar product is negative. Hence the minus sign in eqn (4c) is needed to cancel the negative result of the scalar product to make sure we obtain a positive result for the amount of matter that was in region I. (We can not have a negative matter)

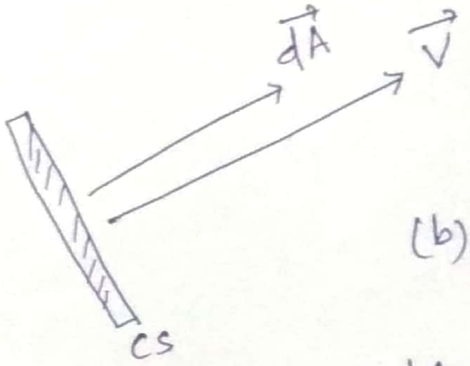
This concept of the sign of the scalar product is illustrated in Fig (3) for (a) the general case of an inlet or exit,



$\vec{V} \cdot \vec{dA} = V\,dA\cos\alpha$ (a) General inlet/exit

(b) an exit velocity parallel to the surface normal

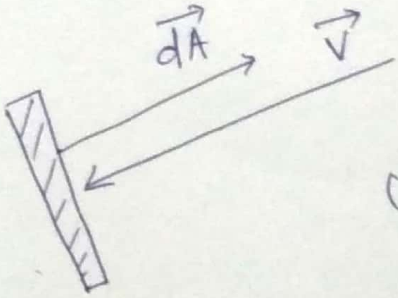
$\cos 0 = 1$



(b) Normal exit

$\vec{V} \cdot \vec{dA} = +V\,dA$

(c) an inlet velocity parallel to the surface normal



(c) Normal inlet

$\cos 180 = -1$

$\vec{V} \cdot \vec{dA} = -V\,dA$

Evaluating the scalar Product

Cases (b) and (c) are obviously convenient special case of (a); the value of the cosine in case (a) automatically generates the correct sign of either an inlet or an exit.

We can finally use eqn 4(a), 4(b) and 4(c) in eqn (3) to obtain

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, dV + \int_{CS_I} \eta \rho \vec{V} \cdot \vec{dA} + \int_{CS_{III}} \eta \rho \vec{V} \cdot \vec{dA}$$

and the two last integral can be ~~combined~~ combined because CS_I and CS_{III} constitute the entire control surface

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, dV + \int_{CS} \eta \rho \vec{V} \cdot \vec{dA} \quad (5)$$

RTT

Eqn (5) is a relation we set out to obtain.
It is the fundamental relation between the rate of change of any arbitrary extensive property, N of a system and the variations of this property associated with a control volume.

Some authors refer to eqn (5) as the Reylonds Transport Theorem

Physical Interpretation

We ~~k~~ now have a formula. (Reynolds Transport Theorem)

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, dV + \int_{CS} \eta \rho \vec{V}\cdot d\vec{A}$$

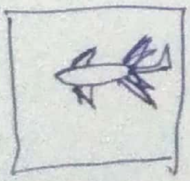
⇒ This formula (RTT) can be used to convert the rate of change of any extensive property N of a system to an equivalent formulation for use with a control volume.

⇒ ~~This RTT can be used~~ We can now use RTT in the various basic physical law equations one by one, with N replaced with each of the properties $M, \vec{P}, \vec{H}$, E & S (with corresponding symbol η) to replace system derivatives with control volume expressions.

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, dV + \int_{CS} \eta \rho \vec{V}\cdot d\vec{A}$$

⇒ the system is the matter that happens to be passing through the chosen control volume, at the instant we chose.

⇒ For example, if we chose as a control volume the region contained by an airplane wing and an imaginary rectangular boundary around it, the system would be the mass of air that is instantaneously contained between the rectangle and the airfoil.



Let us discuss the meaning of each term.

$\left.\frac{dN}{dt}\right)_{system}$ → is the rate of change of system extensive property N.
For example, if $N = \vec{P}$, we obtained the rate of change of momentum.

$\frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall$ → is the rate of change of the amount of property N in the control volume.

→ This term computes the instantaneous value of N in the control volume

— ($\int_{CV} \rho \, d\forall$ is the instantaneous mass in the control volume)

— For example if $N = \vec{P}$, $\eta = \vec{V}$ and $\int_{CV} \vec{V} \rho \, d\forall$ computes the instantaneous amount of momentum in the control volume.

$\int_{cs} \eta \rho \vec{V} \cdot d\vec{A}$ → is the rate at which property N is exiting the surface of the control volume.

→ The term $\rho \vec{V} \cdot d\vec{A}$ computes the rate of mass transfer leaving across control surface area element $d\vec{A}$, multiplying by η computes the rate of flux of property N across the element and itegrating therefore computes the net flux of N out of the control volume

For example if $N = \vec{P}$ and $\eta = \vec{V}$

and $\int_{cs} \vec{V} \rho \vec{V} \cdot d\vec{A}$ computes the net flux of momentum out of the control volume.

---

* Care should be taken in evaluating the dot product: Because $\vec{A}$ is always directed outwards, the ~~pos~~ dot product will be positive when $\vec{V}$ is outward and negative when $\vec{V}$ is inward, $\vec{V}$ is measured with respect to C.V.

Conservation of Mass :-

⇒ The first physical principle to which we apply this conversion from a system to a control volume description is the mass conservation principle :-

The mass of the system remains constant,

$$\left.\frac{dM}{dt}\right)_{system} = 0 \quad —①$$

where

$$M_{system} = \int_{M(system)} dm = \int_{\forall(system)} \rho \, d\forall \quad —②$$

The system and control volume formulations are related by

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{cv} \eta \rho \, d\forall + \int_{cs} \eta \rho \vec{V}\cdot d\vec{A} \quad —③$$

where $N_{system} = \int_{M(system)} \eta \, dm = \int_{\forall(system)} \eta \rho \, d\forall \quad —④$

To derive the control volume formulation of conservation of mass, we set

$N = M$ and $\eta = 1$

With this substitution, we obtain

$$\left.\frac{dM}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V}\cdot d\vec{A} \quad —⑤$$

②

Compairing eqⁿ ① and eqⁿ ⑤ we arrive at the control volume formulation of the conservation of mass:

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A} = 0 \qquad -(6)$$

In eqⁿ ⑥ the first term represent the rate of change of mass within the control volume.

The second term represent the net rate of mass flux out through the control surface.

Equation ⑥ indicates that the rate of change of mass in control volume plus the net outflow is zero.

The mass conservation equation is also called the continuity equation.

In common-sense terms, the rate of increse of mass in the control volume is due to net inflow of the mass:

Rate of increase of mass in C.V. = Net influx of mass.

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall = -\int_{CS} \rho \vec{V}\cdot d\vec{A}$$

In equation (6) care should be taken in evaluating the scalar product $\vec{V} \cdot d\vec{A} = V dA \cos\alpha$.

It could be positive (outflow $\alpha < \pi/2$)

negative (inflow $\alpha > \pi/2$)

zero ($\alpha = \pi/2$)

$\alpha = 0$ $\qquad$ $\alpha = \pi$

Special Cases

In special cases it is possible to simplyfy equation (6).

⇒ Consider first the case of an incompressible fluid, in which the density remains constant.

⇒ When ρ is constant it is not a function of space and time.

For incompressible fluid equation (6) may be written as

$$\rho \frac{\partial}{\partial t} \int_{CV} d\forall + \rho \int_{CS} \vec{V} \cdot d\vec{A} = 0$$

The integral of $d\forall$ over the control volume is simply the volume of the control volume. Thus, on dividing throughout by ρ, we write

$$\frac{\partial \forall}{\partial t} + \int_{CS} \vec{V} \cdot d\vec{A} = 0$$

⇒ For nondeformable control volume of fixed size and shape,

$\forall$ = constant.

The conservation of mass for incompressible flow through a fixed control volume becomes

$$\int_{CS} \vec{V} \cdot \vec{dA} = 0 \quad \text{— (7a)}$$

A ~~usu~~ useful special case is when we have (or can approximate) uniform velocity at each inlet and exit.

In this case eqn (7a) simplifies to

$$\sum_{CS} \vec{V} \cdot \vec{A} = 0 \quad \text{—— (7b)}$$

Note that we have not assumed the flow to be steady in reducing eqn (6) to (7a) and (7b).

We have only imposed the restriction of incompressible fluid. Thus equation (7a) & (7b) are statement of conservation of mass for flow of an incompressible fluid that may be steady or unsteady.

The dimension of the integrand in equation (7a) are $\frac{L^3}{t}$.

→ The integral $\vec{V} \cdot d\vec{A}$ over a section of the control surface is commonly called the volume flow rate or volume rate of flow.

→ Thus, for incompressible flow, the volume flow rate into a fixed control volume must be equal to the volume flow rate out of the control volume.

→ The volume flow rate Q, through a section of a control surface of area A, is given by

$$Q = \int_A \vec{V} \cdot d\vec{A} \qquad \text{— (8a)}$$

The average velocity magnitude, $\vec{V}$ at a section is defined as

$$\vec{V} = \frac{Q}{A} = \frac{1}{A} \int_A \vec{V} \cdot d\vec{A}$$

(6)

→ Consider now the general case of steady, compressible flow through a fixed control volume.

→ Since the flow is steady, this means that at most $\rho = \rho(x, y, z)$

→ No fluid properties varies with time in a steady flow.

→ The First term of equation (6) must be zero and hence, for steady flow, the statement of conservation of mass reduces to

$$\boxed{\int_{CS} \rho \vec{V} \cdot d\vec{A} = 0} \quad \text{— (9a)}$$

→ A useful special case is when we have uniform velocity at each inlet and exit. Eqn (9a) simplifies to

$$\boxed{\sum_{CS} \rho \vec{V} \cdot \vec{A} = 0} \quad \text{— (9b)}$$

Thus, for steady flow rate, the mass flow rate into a control volume must be equal to the mass flow rate out of the control volume.

Ex 4.1 →	uniform flow rate at each section
Ex 4.2 →	do not have uniform flow
Ex 4.3 →	Unsteady flow

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Module II (Lec 4)

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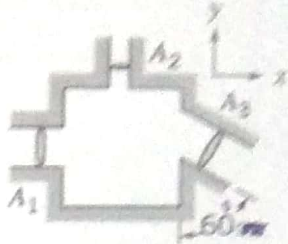
Exercise

Problem 4.24 Page 151

[Difficulty: 1]

4.24 Fluid with ~~65 lbm/ft³~~ 1040 kg/m³ density is flowing steadily through the rectangular box shown. Given $A_1 = 0.5\ \text{ft}^2$, $A_2 = 0.1\ \text{ft}^2$, $A_3 = 0.6\ \text{ft}^2$, $\vec{V}_1 = 10\hat{i}\ \text{ft/s}$, and $\vec{V}_2 = 20\hat{j}\ \text{ft/s}$, determine velocity $\vec{V}_3$.



$A_1 = .046\ m^2$ $A_2 = .009\ m^2$ $A_3 = .056\ m^2$

$V_1 = 3\hat{i}\ m/s$ $\vec{V}_2 = 6\hat{j}\ m/s$.

Given: Data on flow through box

Find: Velocity at station 3

Solution:

Basic equation $\sum_{CS} \left(\vec{V}\cdot\vec{A}\right) = 0$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Then for the box $\sum_{CS} \left(\vec{V}\cdot\vec{A}\right) = -V_1 \cdot A_1 + V_2 \cdot A_2 + V_3 \cdot A_3 = 0$

Note that the vectors indicate that flow is in at location 1 and out at location 2; we assume outflow at location 3

Hence $V_3 = V_1 \cdot \frac{A_1}{A_3} - V_2 \cdot \frac{A_2}{A_3}$ $V_3 = 10 \cdot \frac{ft}{s} \times \frac{0.5}{0.6} - 20 \cdot \frac{ft}{s} \times \frac{0.1}{0.6}$ $V_3 = 5 \cdot \frac{ft}{s}$

Based on geometry $V_x = V_3 \cdot \sin(60 \cdot deg)$ $V_x = 4.33 \cdot \frac{ft}{s}$

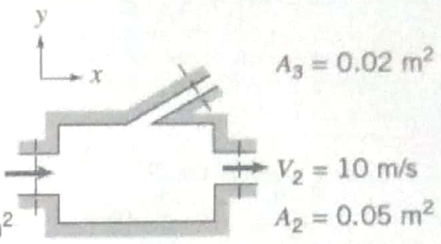
$V_y = -V_3 \cdot \cos(60 \cdot deg)$ $V_y = -2.5 \cdot \frac{ft}{s}$

$\vec{V_3} = \left(4.33 \cdot \frac{ft}{s}, -2.5 \cdot \frac{ft}{s}\right)$

Problem 4.25

[Difficulty: 1]

4.25 Consider steady, incompressible flow through the device shown. Determine the magnitude and direction of the volume flow rate through port 3.



Given: Data on flow through device

Find: Volume flow rate at port 3

Solution:

Basic equation
$$\sum_{CS} \left(\vec{V} \cdot \vec{A}\right) = 0$$

Assumptions: 1) Steady flow 2) Incompressible flow 3) Uniform flow

Then for the box
$$\sum_{CS} \left(\vec{V} \cdot \vec{A}\right) = -V_1 \cdot A_1 + V_2 \cdot A_2 + V_3 \cdot A_3 = -V_1 \cdot A_1 + V_2 \cdot A_2 + Q_3$$

Note we assume outflow at port 3

Hence $Q_3 = V_1 \cdot A_1 - V_2 \cdot A_2$ $\quad Q_3 = 3 \cdot \frac{m}{s} \times 0.1 \cdot m^2 - 10 \cdot \frac{m}{s} \times 0.05 \cdot m^2 \quad Q_3 = -0.2 \cdot \frac{m^3}{s}$

The negative sign indicates the flow at port 3 is inwards. Flow rate at port 3 is 0.2 m^3/s inwards

Momentum equation for Inertial Control Volume:-

→ We now wish to obtain a control volume form of Newton's second law.

→ We use the same procedure we just used for mass conservation, with one note of caution:

⇒ the control volume coordinates (with respect to which we measure all velocities) are inertial (i.e) the control volume coordinates xyz are either at rest or moving at constant speed with respect to an "absolute" set of co-ordinates XYZ.

⇒ We begin with mathematical formulation for a system and then use RTT to go from the system to the control volume formulation.

⇒ Newton's second law for a system moving relative to inertial co-ordinate system is given by equation

$$\vec{F} = \left.\frac{d\vec{P}}{dt}\right)_{system} \quad -①$$

where the linear momentum of a system is given by

$$\vec{P}_{system} = \int_{M(system)} \vec{V}\, dm = \int_{\forall(system)} \vec{V} \rho\, d\forall \quad -②$$

The resultant force, $\vec{F}$ includes all surface and the body forces acting on the system,

$$\vec{F} = \vec{F_S} + \vec{F_B}$$

The system and control volume formulations are related using

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall + \int_{CS} \eta \rho \vec{V} \cdot d\vec{A}$$

To derive the control volume formulation of Newton's second law, we set

$$N = \vec{P} \quad \text{and} \quad \eta = \vec{V}$$

So,

$$\left.\frac{d\vec{P}}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \vec{V} \cdot d\vec{A} \quad \text{—(3)}$$

From eq^n (1)

$$\left.\frac{d\vec{P}}{dt}\right)_{system} = \vec{F}\Big)_{on\ system.}$$

In deriving RTT, the system and control volume coincide at t_0, then.

$$\vec{F}\Big)_{on\ system} = \vec{F}\Big)_{on\ control\ volume.}$$

In light of this, eqⁿ ① and eqⁿ ③ may be combined to yield the control volume formulation of Newton's second law for a nonaccelerating control volume

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\,\rho\, d\forall + \int_{CS} \vec{V}\,\rho\vec{V}\cdot d\vec{A} \quad \text{— (4a)}$$

For cases when we have uniform flow at each inlet and exit, we can use

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\,\rho\, d\forall + \sum_{CS} \vec{V}\,\rho\vec{V}\cdot\vec{A} \quad \text{— (4.b)}$$

Eqⁿ (4a) and eqⁿ (4b) are our (nonaccelerating) control volume forms of Newton's second law.

It states that the total force (due to surface and body forces) acting on the control volume leads to rate of change of momentum within the control volume (the volume integral) and a net rate at which momentum is leaving the control volume through the control surface.

We must be little careful in applying eqⁿ (4).

→ The First step will always be to carefully choose a control volume and its control surface so that we can evaluate the volume integral and surface integral (or summation)

→ Each inlet and exit should be carefully labeled as should the external forces acting.

In fluid mechanics the body force is usually gravity so.

$$\vec{F}_B = \int_{CV} \rho \vec{g}\, dV = \vec{W}_{CV} = M\vec{g}$$

where $\vec{g}$ is the acceleration of gravity and $\vec{W}_{CV}$ is the instantaneous weight of the entire control volume

In many application the surface force is due to pressure

$$\vec{F}_s = \int_A -p\, d\vec{A}$$

Note :- The -ve sign (minus sign) is to ensure that we always compute pressure forces acting onto the control surface
→ ($d\vec{A}$ was chosen to be a vector pointing out of the control volume)
→ (It is worth stressing that even at a points on the surface that have an outflow, the pressure force acts onto the control volume)

⑤

In eqn ④ we must also be careful in evaluating

$$\int_{cs} \vec{V} \rho \vec{V} \cdot dA \quad \text{or} \quad \sum_{cs} \vec{V} \rho \vec{V} \cdot \vec{A}$$

(this may be easier to do if we write them with the implied parentheses $\int_{cs} \vec{V} \rho (\vec{V} \cdot dA)$ or $\sum_{cs} \vec{V} \rho (\vec{V} \cdot \vec{A})$)

The velocity $\vec{V}$ must be measured with respect to control volume co-ordinates xyz, with the ~~co-ordinat~~ appropriate signs for its vector components u, v and w, recall also that the scalar product will be positive for outflow and negative for inflow.

The momentum equation (4) is a vector equation. We will usually write the three scalar components, as measured in the xyz co-ordinates of the control volume,

$$F_x = F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{cv} u \rho \, d\forall + \int_{cs} u \rho \vec{V} \cdot d\vec{A} \quad \text{— (5a)}$$

$$F_y = F_{Sy} + F_{By} = \frac{\partial}{\partial t}\int_{cv} v \rho \, d\forall + \int_{cs} v \rho \vec{V} \cdot d\vec{A} \quad \text{— (5b)}$$

$$F_z = F_{Sz} + F_{Bz} = \frac{\partial}{\partial t}\int w \rho \, d\forall + \int w \rho \vec{V} \cdot d\vec{A} \quad \text{— (5c)}$$

For uniform flow at each inlet and outlet

$$+ \sum_{cs} u \rho \vec{V} \cdot d\vec{A}$$

$$+ \sum_{cs} v \rho \vec{V} \cdot d\vec{A}$$

$$+ \sum_{cs} w \rho \vec{V} \cdot d\vec{A}$$

— ⑥

Control Volume Moving with Constant Velocity

→ We studied the applications of momentum equation to inertial control volumes (stationary control volume.

→ suppose we have a control volume moving at constant speed.

→ We can set up two co-ordinate systems

(i) XYZ, absolute or stationary (inertial) co-ordinate

(ii) the xyz co-ordinates attached to the control volume (also inertial because the control volume is not accelerating with respect to XYZ)

→ The RTT which expresses system derivatives in terms of control volume variable is valid for any motion of the control volume provided that co-ordinate system xyz are measured relatives to the all velocities are control volume.

So we ~~have~~ can rewrite

$$\left.\frac{dN}{dt}\right)_{system} = \frac{\partial}{\partial t}\int_{CV} \eta \rho \, d\forall + \int_{CS} \eta \rho \, \vec{V}_{xyz} \cdot d\vec{A}$$

—(7)

→ Since all velocities must be measured relative to the control volume, in using this equation to obtain the momentum equation for an inertial control volume equation from the system formulation, we must set

$$N = \vec{P}_{xyz} \quad \text{and} \quad \eta = \vec{V}_{xyz}$$

The control volume equation is then written as

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\, \rho\, d\forall + \int_{CS} \vec{V}_{xyz}\, \rho\, \vec{V}_{xyz} \cdot d\vec{A} \qquad - (8)$$

Eqn (8) is the formulation of Newton's second law applied to any inertial control volume (stationary or moving with a constant velocity).

→ We have used/included subscript xyz to emphasize that velocities must be measured relative to the control volume

Lecture I (Module II, ch 4)

Differential Analysis to Fluid Motion

→ We developed the basic equations in integral form for a control volume.

→ Integral equations are useful when we are interested in the gross behavior of a flowfield and its effect on various devices

→ However the integral approach does not enable us to obtain detailed point by point information/knowledge of the flow field.

→ For example, the integral approach could provide information on the lift generated by a wing.

→ It could not be used to determine the pressure distribution that produce the lift on the wing.

* → In order to find out the flow information in detail, we need differential form of the equations of motion.

→ In this chapter we shall develop the differential equations for the conservation of mass and Newton's second law of motion

We are interested in developing differential equations, we need to analyze infinitesimal systems and control volumes.

Conservation of Mass

→ We studied that the property fields are defined by continuous functions of space coordinates and time.

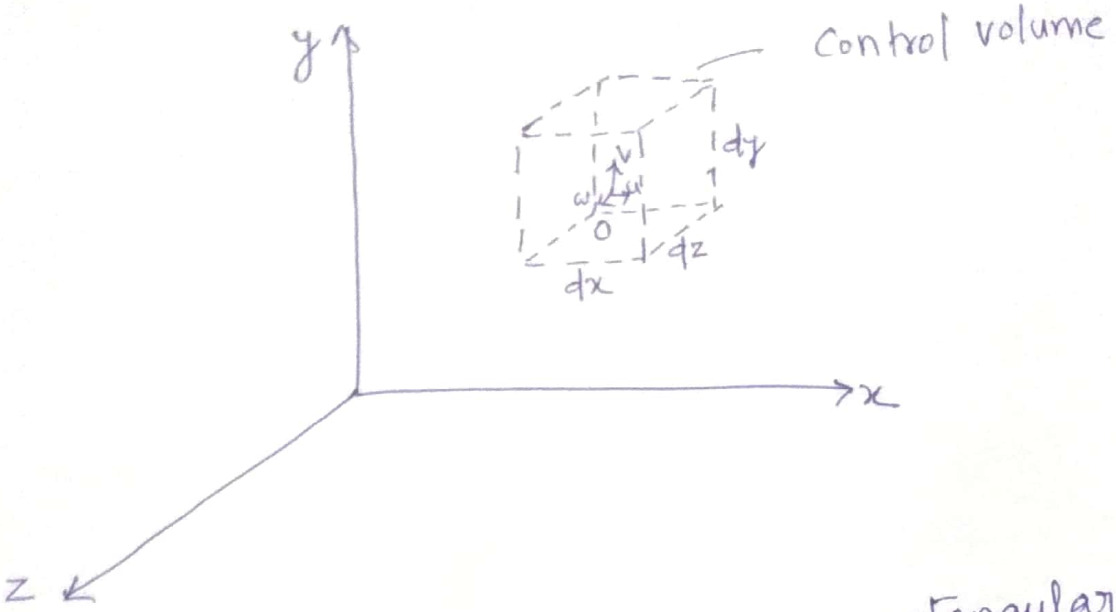
→ The density and velocity fields are related through conservation of mass in integral form of control volume representation.

→ We shall derive the differential equation for conservation of mass in rectangular co-ordinates.

→ In ~~both~~ this cases the derivation is carried out by applying conservation of mass to a differential control volume.

(3)

Rectangular Coordinate System



(Differential Control volume in rectangular co-ordinates)

→ The control volume chosen is an infinitesimal cube with sides of length dx, dy, dz as shown in figure.

→ The density at the center O, of the control volume is assumed to be ρ and the velocity there is assumed to be

$$\vec{V} = \hat{i}u + \hat{j}v + \hat{k}w$$

→ To evaluate the properties at each of the six faces of the control surface, we use ~~Ted~~ Taylor series expansion about point O.

For example at the right face,

$$\rho\Big)_{x+\frac{dx}{2}} = \rho + \left(\frac{\partial \rho}{\partial x}\right)\frac{dx}{2} + \left(\frac{\partial^2 \rho}{\partial x^2}\right)\frac{1}{2!}\left(\frac{dx}{2}\right)^2 + $$

Neglecting higher order terms, we can write

$$\rho\Big)_{x+\frac{dx}{2}} = \rho + \left(\frac{\partial \rho}{\partial x}\right)\frac{dx}{2}$$

Similarly

$$u\Big)_{x+\frac{dx}{2}} = u + \left(\frac{\partial u}{\partial x}\right)\frac{dx}{2}$$

where $\rho, u, \frac{\partial \rho}{\partial x}$, and $\frac{\partial u}{\partial x}$ are evaluated at point O.

The corresponding terms at the left face are

$$\rho\Big)_{x-\frac{dx}{2}} = \rho + \left(\frac{\partial \rho}{\partial x}\right)\left(-\frac{dx}{2}\right) = \rho - \left(\frac{\partial \rho}{\partial x}\right)\frac{dx}{2}$$

$$u\Big)_{x-\frac{dx}{2}} = u + \left(\frac{\partial u}{\partial x}\right)\left(-\frac{dx}{2}\right) = u - \left(\frac{\partial u}{\partial x}\right)\frac{dx}{2}$$

We can write similar expression involving ρ and v for the front and back faces.

for the front and back faces of the ρ and w for the top and bottom faces of the infinitesimal cube $dx\,dy\,dz$.

These can then be used to evaluate the surface integral in eqⁿ (1)

$$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0 \quad -(1)$$

($\int_{CS} \rho \vec{V} \cdot d\vec{A}$ is the net flux of mass out of the control volume)

~~$\frac{\partial}{\partial t}\int_{CV} \rho \, d\forall + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$~~

Mass Flux through the Control Surface of a Rectangular Differential control volume

Surface	Evaluation of $\int \rho \vec{V} \cdot d\vec{A}$
Left $(-x)$	$= -\left[\rho - \left(\frac{\partial \rho}{\partial x}\right)\frac{dx}{2}\right]\left[u - \left(\frac{\partial u}{\partial x}\right)\frac{dx}{2}\right] dy\,dz$ $= -\rho u\,dy\,dz + \frac{1}{2}\left[u\left(\frac{\partial \rho}{\partial x}\right) + \rho\left(\frac{\partial u}{\partial x}\right)\right] dx\,dy\,dz$
Right $(+x)$	$= \left[\rho + \left(\frac{\partial \rho}{\partial x}\right)\frac{dx}{2}\right]\left[u + \left(\frac{\partial u}{\partial x}\right)\frac{dx}{2}\right] dy\,dz$ $= \rho u\,dy\,dz + \frac{1}{2}\left[u\left(\frac{\partial \rho}{\partial x}\right) + \rho\left(\frac{\partial u}{\partial x}\right)\right] dx\,dy\,dz$
Bottom $(-y)$	$= -\left[\rho - \left(\frac{\partial \rho}{\partial y}\right)\frac{dy}{2}\right]\left[v - \left(\frac{\partial v}{\partial y}\right)\frac{dy}{2}\right] dx\,dz$ $= -\rho v\,dx\,dz + \frac{1}{2}\left[v\left(\frac{\partial \rho}{\partial y}\right) + \rho\left(\frac{\partial v}{\partial y}\right)\right] dx\,dy\,dz$
Top $(+y)$	$= \left[\rho + \left(\frac{\partial \rho}{\partial y}\right)\frac{dy}{2}\right]\left[v + \left(\frac{dv}{dy}\right)\frac{dy}{2}\right] dx\,dz$ $= \rho v\,dx\,dz + \frac{1}{2}\left[v\left(\frac{d\rho}{dy}\right) + \rho\left(\frac{\partial v}{\partial y}\right)\right] dx\,dy\,dz$

Back (−z) $= -\left[\rho - \left(\frac{\partial \rho}{\partial z}\right)\frac{dz}{2}\right]\left[w - \left(\frac{\partial w}{\partial z}\right)\frac{dz}{2}\right] dx\,dy$

$= -\rho w\,dx\,dy + \frac{1}{2}\left[w\left(\frac{\partial \rho}{\partial z}\right) + \rho\left(\frac{\partial w}{\partial z}\right)\right] dx\,dy\,dz$

Front (+z) $= \left[\rho + \left(\frac{\partial \rho}{\partial z}\right)\frac{dz}{2}\right]\left[w + \left(\frac{\partial w}{\partial z}\right)\frac{dz}{2}\right] dx\,dy$

$= \rho w\,dx\,dy + \frac{1}{2}\left[w\left(\frac{\partial \rho}{\partial z}\right) + \rho\left(\frac{\partial w}{\partial z}\right)\right] dx\,dy\,dz$

Adding the results for all six faces.

$$\int_{cs} \rho \vec{V}\cdot d\vec{A} = \left[\left\{u\left(\frac{\partial \rho}{\partial x}\right) + \rho\left(\frac{\partial u}{\partial x}\right)\right\} + \left\{v\left(\frac{\partial \rho}{\partial y}\right) + \rho\left(\frac{\partial v}{\partial y}\right)\right\} + \left\{w\left(\frac{\partial \rho}{\partial z}\right) + \rho\left(\frac{\partial w}{\partial z}\right)\right\}\right] dx\,dy\,dz$$

$$\Rightarrow \int_{cs} \rho \vec{V}\cdot d\vec{A} = \left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}\right] dx\,dy\,dz$$

→ We assume that the velocity components u, v, w are positive in the x, y and z directions respectively.

→ the area normal is by convention positive out of the cube and higher order terms are neglected in the limit as dx, dy and dz → 0

The result of all this work is

$$\left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial x} + \frac{\partial \rho w}{\partial x}\right] dx\, dy\, dz$$

This expression is the surface integral evaluation for our differential cube.

To complete eqⁿ (1), we need to evaluate the volume integral.

$$\left[\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall \rightarrow \text{the rate of change of mass in the control volume}\right]$$

$$\frac{\partial}{\partial t}\int_{cv} \rho\, d\forall \rightarrow \frac{\partial}{\partial t}\left[\rho\, dx\, dy\, dz\right]$$

$$= \frac{\partial \rho}{\partial t} dx\, dy\, dz$$

We obtain (after canceling $dx\, dy\, dz$) from eqⁿ (1) a differential form of the mass conservation ~~law~~ law

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \quad \text{— (2a)}$$

Equation (2a) is frequently called the continuity equation.

Since the vector operator, ∇, in rectangular co-ordinate is given by

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

then.

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = \nabla \cdot \rho \vec{V}$$

Note that the del operator ∇ acts on ρ and $\vec{V}$.

Think of it as $\nabla \cdot (\rho \vec{V})$.

The conservation of mass may be written as

$$\boxed{\nabla \cdot \rho \vec{V} + \frac{\partial \rho}{\partial t} = 0} \quad \text{— ~~(2b)~~ (2b)}$$

Two cases

(i) For an incompressible fluid $\rho =$ constant density is neither a function of space co-ordinate nor a function of time.

For an incompressible, the continuity equation simplifies to

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot \vec{V} = 0} \quad \text{— 2(c)}$$

(ii) For steady flow, all fluid properties are by defination, independent of time. So the continuity eqn

Thus $\frac{\partial \rho}{\partial t} = 0$ and $\rho = \rho(x, y, z)$

$$\boxed{\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = \nabla \cdot \rho \vec{V} = 0} \quad \text{— (2d)}$$

del operator ∇ acts on ρ and $\vec{V}$.

(9)

Momentum Equation for Control Volume with acceleration (Rectilinear) (c.v. accelerating without rotation)

$$\vec{F} = \vec{F}_s + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\, \rho\, d\forall + \int_{CS} \vec{V}_{xyz}\, \rho\, \vec{V}_{xyz} \cdot d\vec{A}$$

$$\vec{F}_S + \vec{F}_B - \int_{CV} \vec{a}_{rf}\, \rho\, d\forall = \frac{\partial}{\partial t}\int_{CV} \vec{V}_{xyz}\, \rho\, d\forall + \int_{CS} \vec{V}_{xyz}\, \rho\, \vec{V}_{xyz} \cdot d\vec{A}$$

For non accelerating $a_{rf} = 0$

$a_{rf} \rightarrow$ rectalinear acceleration of noninertial reference frame xyz (i.e. of the control volume) relative to the inertial frame XYZ.

Angular Momentum Principle : (For inertial control volume)

$$\vec{T} = \left.\frac{d\vec{H}}{dt}\right)_{system}$$

$$\vec{r} \times \vec{F}_s + \int_{CV} \vec{r} \times \vec{g}\, \rho\, d\forall + T_{shaft}$$
$$= \frac{\partial}{\partial t}\int_{CV} \vec{r} \times \vec{V}\, \rho\, d\forall + \int_{CS} \vec{r} \times \vec{V}\, \rho\, \vec{V} \cdot d\vec{A}$$

Ex 5.1

For a 2D flow in the xy plane, the x component of velocity is given by $u = Ax$. Determine a possible y component for incompressible flow. How many y components are possible?

Given 2D flow in xy plane for which $u = Ax$

Find: (a) Possible y component for incompressible flow.

(b) Number of possible y-components.

5.9 The x component of velocity in a steady incompressible flow field in the xy plane is $u = Ae^{x/b}\cos(y/b)$, where $A = 10$ m/s, $b = 5$ m, and x and y are measured in meters. Find the simplest y component of velocity for this flow field.

Given: x component of velocity

Find: y component for incompressible flow; Valid for unsteady? How many y components?

Solution:

Basic equation: $\frac{\partial}{\partial x}(\rho \cdot u) + \frac{\partial}{\partial y}(\rho \cdot v) + \frac{\partial}{\partial z}(\rho \cdot w) + \frac{\partial}{\partial t}\rho = 0$

Assumption: Incompressible flow; flow in x-y plane

Hence $\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = 0$ or $\frac{\partial}{\partial y}v = -\frac{\partial}{\partial x}u = -\frac{\partial}{\partial x}\left(A \cdot e^{\frac{x}{b}} \cdot \cos\left(\frac{y}{b}\right)\right) = -\left(\frac{A}{b} \cdot e^{\frac{x}{b}} \cdot \cos\left(\frac{y}{b}\right)\right)$

Integrating $v(x,y) = -\int \frac{A}{b} \cdot e^{\frac{x}{b}} \cdot \cos\left(\frac{y}{b}\right) dy = -A \cdot e^{\frac{x}{b}} \cdot \sin\left(\frac{y}{b}\right) + f(x)$

This basic equation is valid for steady and unsteady flow (t is not explicit)

There are an infinite number of solutions, since f(x) can be any function of x. The simplest is f(x) = 0

$$v(x,y) = -A \cdot e^{\frac{x}{b}} \cdot \sin\left(\frac{y}{b}\right) \qquad v(x,y) = -10 \cdot e^{\frac{x}{5}} \cdot \sin\left(\frac{y}{5}\right)$$

Stream Function for Two-Dimensional Incompressible Flow :-

→ Streamlines are lines drawn in the flow field so that at a given instant they are tangent to the direction of flow at every point in the flow field.

→ Since streamlines are tangent to the velocity vector at every point in the flow field, there can be no flow across a streamline.

$$\left.\frac{dy}{dx}\right|_{\text{streamlines}} = \frac{v}{u} \quad — ①$$

→ We can now develop a more formal definition of streamlines by introducing the stream function ψ.

→ This will allow us to represent two entities — the velocity components $u(x,y,t)$ and $v(x,y,t)$ of the two-dimensional incompressible flow with a single function $\psi(x,y,t)$

→ There are various ways to define the stream function.

→ We start with the two dimensional version of the continuity equation for incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad — ②$$

We use what looks at first like a purely mathematical exercise (we will see a physical basis for latter) and define function by

$$u \equiv \frac{\partial \psi}{\partial y} \quad \text{and} \quad v \equiv -\frac{\partial \psi}{\partial x} \quad \text{--- (3)}$$

So that equation (2) is automatically satisfied for any $\psi(x, y, t)$ we choose.

We use eqn (2) and eqn (3)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

Using eqn (1), we can obtain an equation valid only along a streamline

$$u\,dy - v\,dx = 0$$

Using the defination of our stream ~~line~~ function.

$$\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \quad \text{--- (4)}$$

On the other hand, from a strict mathematical point of view, at any instant in time t the variation in a function $\psi(x,y,t)$ in space (x, y) is given by

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy \quad \text{—— (5)}$$

Compairing eqn (4) and (5), we see that along an instantaneous streamline, $d\psi = 0$

In other words, ψ is a constant along a streamline.

Hence we can specify individual streamlines by their stream function values

$$\psi = 0, 1, 2, 3 \text{ etc}$$

What is the significance of ψ values?

Ans :- they can be used to obtain the volume flow rate between any two streamlines.

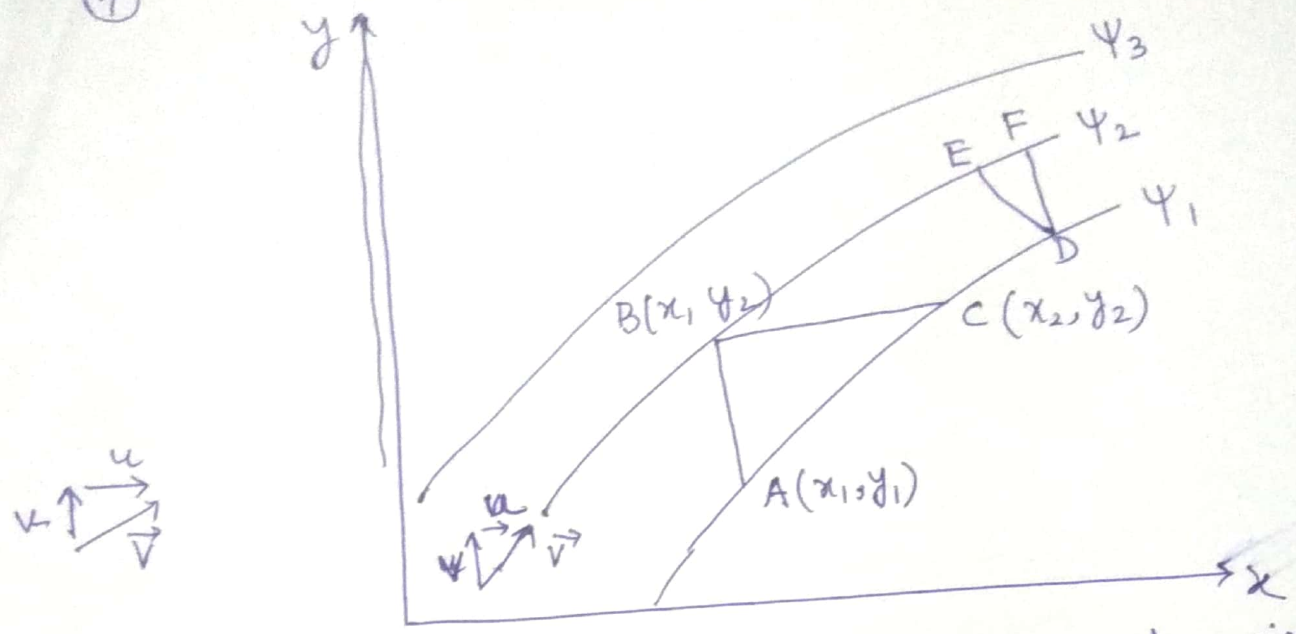


Fig1: Instantaneous streamlines in two-dimensional flow ~~Fig1~~.

→ Consider the streamlines shown in Figure 1.. We can compute the volume flow rate between stream lines ψ_1 and ψ_2 by using line AB, BC DE or EF (recall that there is no flow across a streamline)

→ Let us compute the flow rate by using line AB and also by using line BC – they should be same!

→ For unit depth (dimension perpendicular to the xy plane), the flow rate across AB is

$$Q = \int_{y_1}^{y_2} u\,dy = \int_{y_1}^{y_2} \frac{\partial \psi}{\partial y}\,dy$$

But along AB, $x =$ constant, and eqⁿ (5)

$$d\psi = \frac{\partial \psi}{\partial y} dy$$

$$\therefore \quad Q = \int_{y_1}^{y_2} \frac{\partial \psi}{\partial y} dy = \int_{\psi_1}^{\psi_2} d\psi = \psi_2 - \psi_1$$

For unit depth, the flow rate across BC is

$$Q = \int_{x_1}^{x_2} v\,dx = -\int_{x_1}^{x_2} \frac{\partial \psi}{\partial x} dx$$

Along BC, $y =$ constant and from eqⁿ (5)

$$d\psi = \frac{\partial \psi}{\partial x} dx$$

$$\therefore \quad Q = -\int_{x_1}^{x_2} \frac{\partial \psi}{\partial x} dx = -\int_{\psi_2}^{\psi_1} d\psi$$

$$= \psi_2 - \psi_1$$

Hence, whethere we use line AB or line BC (or for that matter line DE or DF), we find that the volume flow rate (per unit depth) between two streamlines is given by the difference between the two stream function values.

For 2D compressible flow

$$\rho u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \rho v = -\frac{\partial \psi}{\partial x}$$

(6)

→ The deviation for lines AB and BC are the justification for using the stream function defination of eqⁿ (3)

→ If the streamline through the origin is designated $\psi = 0$, then ψ value for any other streamline represent the flow between the origin and that streamline.

→ We are free to select any streamline as the zero streamline because the stream function is defined as a differential, also the flow rate will always be given by a difference of ψ values.

(Useful concept) → Note that because the volume flow between any two streamlines is constant, the velocity will be relatively high wherever the streamlines are close together, and relatively low wherever the streamlines are far apart.

For a 2D incompressible flow in the $r\theta$ plane, conservation of mass

$$\frac{\partial (r V_r)}{\partial r} + \frac{\partial V_\theta}{\partial \theta} = 0$$

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \& \quad V_\theta = -\frac{\partial \psi}{\partial r}.$$

5.26 Determine the family of stream function ψ that will yield the velocity field $\vec{V} = 2y(2x+1)\hat{i} + [x(x+1) - 2y^2]\hat{j}$

Soln (i) Given velocity field
(ii) Stream function ψ

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}[2y(2x+1)] + \frac{\partial}{\partial y}[x(x+1) - 2y^2] = 0$$

$$u = 2y(2x+1) = \frac{\partial \psi}{\partial y} \quad \psi(x,y) = \int 2y(2x+1)\,dy$$
$$= 2xy^2 + y^2 + f(x)$$

$$v = x(x+1) - 2y^2 = \frac{\partial \psi}{\partial x} \quad \psi(x,y) = -\int (x(x+1) - 2y^2)\,dx$$
$$= -\frac{x^3}{3} - \frac{x^2}{2} + 2xy^2 + g(y)$$

Compairing these

$$f(x) = -\frac{x^3}{3} - \frac{x^2}{2} \quad \text{and} \quad g(y) = y^2$$

$$\psi(x,y) = y^2 + 2xy^2 - \frac{x^2}{2} - \frac{x^3}{3}$$

checking

$$u(x,y) = \frac{\partial}{\partial y}\left(y^2 + 2xy^2 - \frac{x^2}{2} - \frac{x^3}{3}\right)$$
$$\rightarrow u(x,y) = 2y + 4xy$$
$$v(x,y) = \frac{\partial}{\partial x}\left(y^2 + 2xy^2 - \frac{x^2}{2} - \frac{x^3}{3}\right)$$
$$\rightarrow v(x,y) = x^2 + x - 2y^2$$

(1)

Lec 4 (ch 4)(Module II)

Motion of a Fluid Particle (Kinematics)

Lecture IV (Module II ch 4)

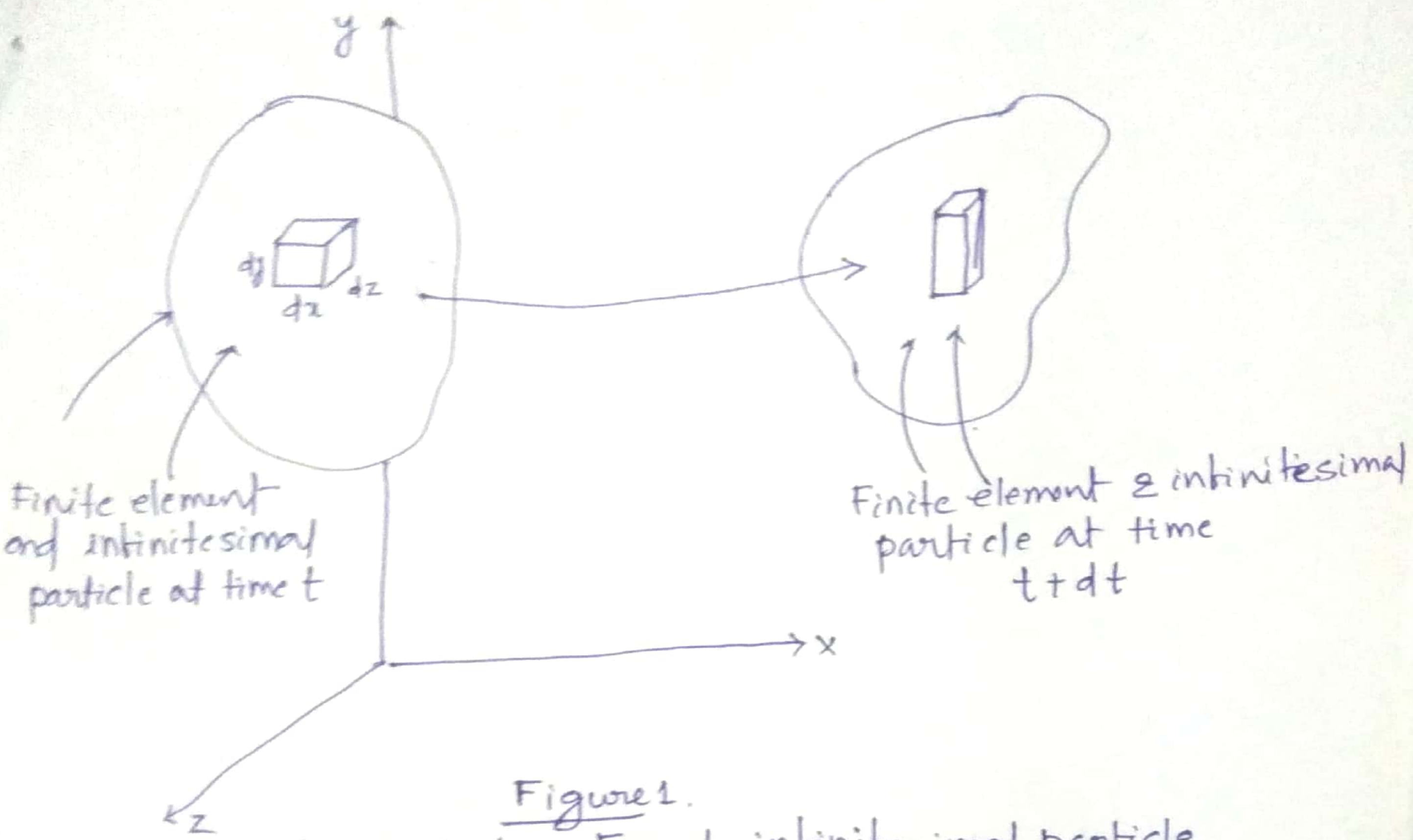


Figure 1.

Finite Fluid element and infinitesimal particle at times t and t+dt

→ Figure 1 shows a typical fluid element, within which we have selected an infinitesimal particle of mass 'dm' and initial volume dx dy dz at time 't' and as it may appear after a time interval dt

→ The finite element has moved and changed its shape and orientation.

→ Note that the finite element has quite severe distortion, the infinitesimal particle has changes ~~its~~ in shape ~~and~~ limited to stretching/shrinking and rotation of the element's side. — this is because we are considering both infinitesimal time step and particle, so that the side remain straight.

→ we will examine the infinitesimal particle so that we will eventually obtain results applicable to a point.

→ We can decompose this particle's motion into four components

→ Translation : in which the particle move from one point to another

→ Rotation :- of the particle, which can occur about any or all of the x, y or z axes.

→ Linear deformation :- in which particle's side stretch or contract

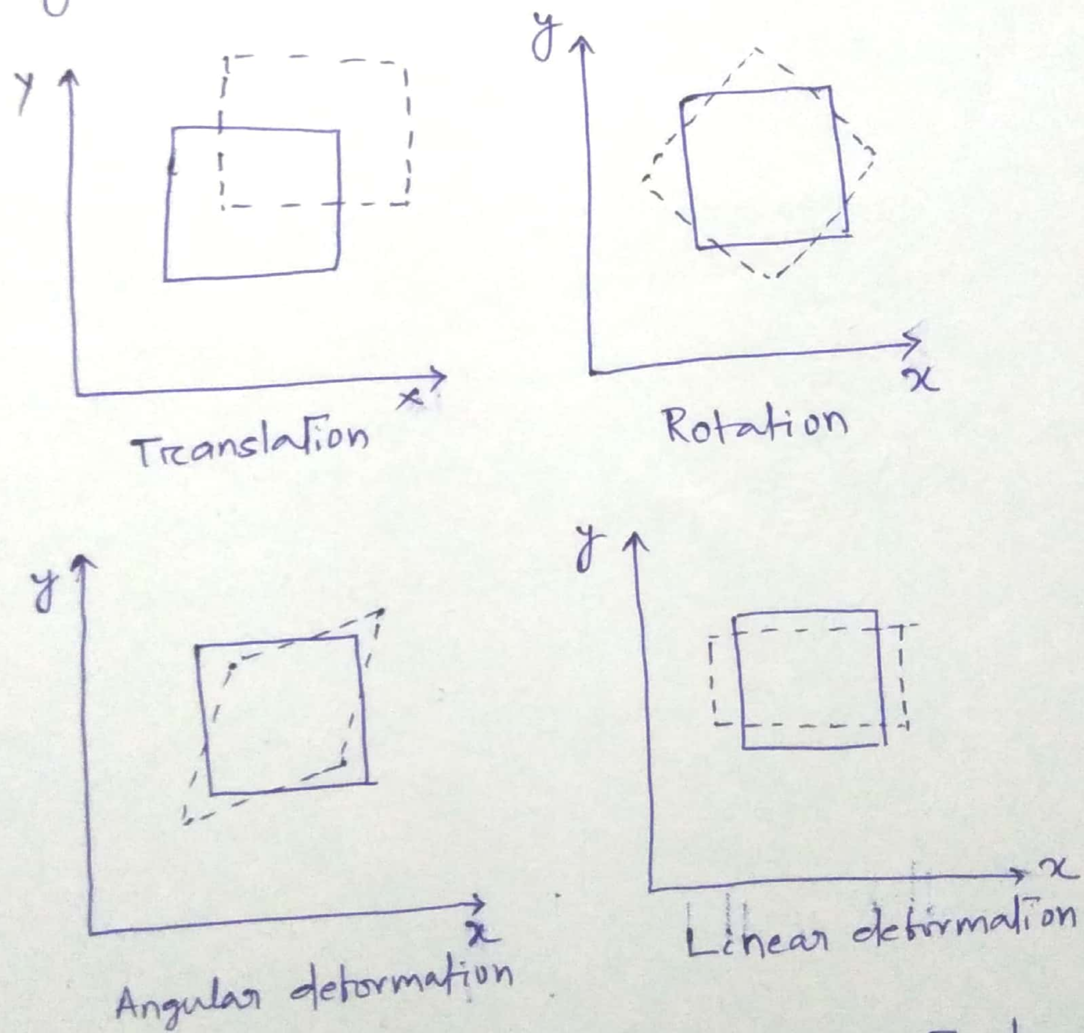
→ Angular Deformation :- in which angles (which was initially 90° for our particle) between the sides changes.

→ It may seem difficult by looking at Figure 1. to distinguish between rotation and angular deformation of the infinitesimal fluid particle.

→ It is important to do so, because pure rotation involves no deformation but angular deformation does

→ <u>we know that fluid deformation generates shear stresses.</u>

→ Figure 2 shows a typical xy plane motion decomposed into four components described above, and we examine each of these four components in turn. we will see that we can distinguish between ~~both~~ rotation ~~ang~~ and angular deformation.



<u>Figure 2</u> Pictorial representation of the components of fluid motion.

(4)

Fluid Translation : Acceleration of a Fluid Particle in a Velocity field.

→ The translation of a fluid particle is connected with the velocity field $\vec{V} = \vec{V}(x, y, z, t)$

→ We will need the acceleration of a fluid particle for use in Newton's second law.

→ It might seem that we could simply compute this as $\vec{a} = \frac{\partial \vec{V}}{\partial t}$.

This is incorrect because $\vec{V}$ is field (ie) it describe the whole flow and not just the motion of a individual particle.

→ The problem, then is to retain the field description for fluid properties and obtain an expression for acceleration of a fluid particle as it moves in a flow field.

→ Given the velocity field $\vec{V} = \vec{V}(x, y, z, t)$, find the acceleration of a fluid particle, $\vec{a_p}$.

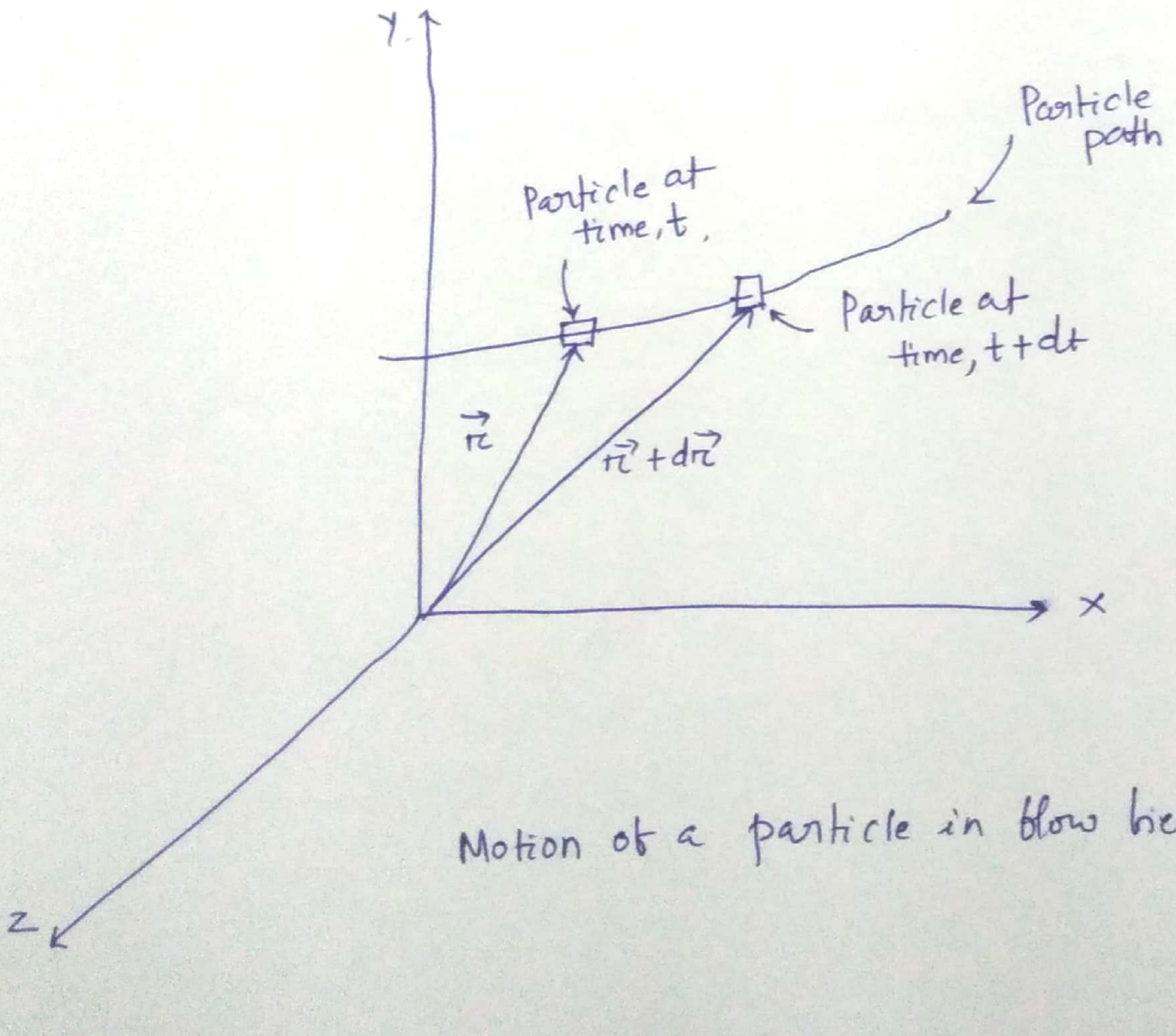
Consider a particle moving in a velocity field

At time t, the particle is at the position x, y, z and has a velocity corresponding to the velocity at that point in a space at time t.

$$\vec{V_P}\Big]_t = \vec{V}(x, y, z, t)$$

At t + dt, the particle has moved to a new position, with co-ordinates x + dx, y + dy, z + dz and has a velocity given by

$$\vec{V_P}\Big]_{t+dt} = \vec{V}(x + dx, y + dy, z + dz, t + dt)$$



Motion of a particle in flow field.

(6)

The particle velocity at time t (position $\vec{r}$) is given by $\vec{V}_p = \vec{V}(x, y, z, t)$

Then $d\vec{V}_p$ the change in velocity of the particle, in moving from location $\vec{r}$ to $\vec{r} + d\vec{r}$ in time 'dt' is given by chain ~~rule~~ rule.

$$d\vec{V}_p = \frac{\partial \vec{V}}{\partial x} dx_p + \frac{\partial \vec{V}}{\partial y} dy_p + \frac{\partial \vec{V}}{\partial z} dz_p + \frac{\partial \vec{V}}{\partial t} dt$$

The total acceleration of the particle is given by

$$\vec{a}_p = \frac{d\vec{V}_p}{dt} = \frac{\partial \vec{V}}{\partial x} \frac{dx_p}{dt} + \frac{\partial \vec{V}}{\partial y} \frac{dy_p}{dt} + \frac{\partial \vec{V}}{\partial z} \frac{dz_p}{dt} + \frac{\partial \vec{V}}{\partial t}$$

since.

$$\frac{dx_p}{dt} = u, \quad \frac{dy_p}{dt} = v, \text{ and } \frac{dz_p}{dt} = w$$

we have.

$$\vec{a}_p = \frac{d\vec{V}_p}{dt} = u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$$

To remind us that calculation of the acceleration of a fluid particle in a velocity field requires a special derivative, it is given the symbol $\frac{D\vec{V}}{Dt}$, Thus

$$\frac{D\vec{V}}{Dt} = \vec{a}_p = u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z} + \frac{\partial \vec{V}}{\partial t}$$

substantial derivative. /material derivative/particle derivative

The physical significance of the terms

$$\vec{a_p} = \frac{D\vec{V}}{Dt} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{convective acceleration}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{local acceleration}}$$

total acceleration of a particle

A fluid particle moving in a flow field may undergo acceleration for either of two reasons.

* In a steady flow in which particles are convected toward the low-velocity region (near corner) and then away to high-velocity region.

If the flow field is unsteady a fluid particle will undergo an additional local acceleration, because velocity is a function of time.

The convective acceleration may be written as a single vector expression using the gradient operator ∇. Thus

$$u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} = (\vec{V}\cdot\nabla)\vec{V}$$

So above eq^n can be written as

$$\frac{D\vec{V}}{Dt} = \vec{a_p} = (\vec{V}\cdot\nabla)\vec{V} + \frac{\partial \vec{V}}{\partial t} \quad \text{— (1)}$$

For 2D flow

$\vec{V} = \vec{V}(x, y, t)$. Eq^n (1) reduces to

$$\frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + \frac{\partial \vec{V}}{\partial t}$$

For 1-D flow $\vec{V} = \vec{V}(x,t)$ Eq^n (1) becomes

$$\frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + \frac{\partial \vec{V}}{\partial t}$$

For a steady flow in three dimensions eqn-① becomes

$$\frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

A fluid particle may undergo a convective acceleration due to its motion, even in a steady velocity field.

Eqn ① is a vector equation.

It may be written in scalar component equations

Relative to an xyz co-ordinate system, the scalar component of eqn ① are written

acceleration of fluid particle anywhere. Eulerian method

$$a_{xp} = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \quad \text{—(2a)}$$

$$a_{yp} = \frac{Dv}{Dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \quad \text{—(2b)}$$

$$a_{zp} = \frac{Dw}{Dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \quad \text{—(2c)}$$

Similarly for cylindrical co-ordinate.

Lagrangian method. } the acc^n, position and velocity of a particle are specified as a function of time only.

(track individual particle)

(1)

Module - III

Incompressible Inviscid flow

→ The differential Momentum equation that describe the behavior of any fluid satisfy the continum assumption.

→ These equations reduced to various particular forms - the most well known being the Navier - Stokes equations, for an incompressible, constant viscosity fluid.

→ The N-S equation describe the behavior of common fluids (eg. water, air, lubricating oil) for a wide range of problems - they are unsolvable analytically except for the simplest of geometries and flows.

→ In this chapter, instead of the Navier-Stokes equations, we will study Euler's equation, which applies to an inviscid fluid.

→ Although truly inviscid fluid do not exist, many flow problems (especially in aerodynamics) can be successfully analyzed with the approximation that $\mu = 0$.

Momentum equation for frictionless flow: Euler's Equation

Euler equation (obtained from N-S equations after neglecting the viscous terms) is

$$\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p \quad \text{—} (1)$$

→ This equation states that for an inviscid fluid the change in momentum of a fluid particle is caused by the body force (assumed to be gravity only) and the net pressure force.

For convenience we recall that the particle acceleration is

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V} \quad \text{—} (2)$$

In addition to the equation (1), we have the incompressible form of the mass conservation equation

$$\nabla\cdot\vec{V} = 0 \quad \text{—} (3)$$

Equation (1) can be expressed in rectangular co-ordinates is

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial p}{\partial x} \quad \text{—} (4a)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial p}{\partial y} \quad \text{—} (4b)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} \quad \text{—} (4c)$$

(3)

If the z axis is assumed vertical, then

$$g_x = 0 \quad g_y = 0 \quad g_z = -g \quad \text{or} \quad \vec{g} = -g\hat{k}.$$

Equations (1) and equations (2) apply to problems in which there are no viscous stresses.

→ Lets consider for a moment when we have no viscous stresses, other than when $\mu = 0$.

→ In general viscous stresses are present when we have fluid deformation. (how we initially defined a fluid)

→ When we have no fluid deformation (i.e) when we have rigid-body motion, no viscous stresses will be present even if $\mu \neq 0$.

→ Hence Euler's equations apply to rigid body motions as well as to inviscid flow.

④

Euler's Equations in Streamline Coordinates :-

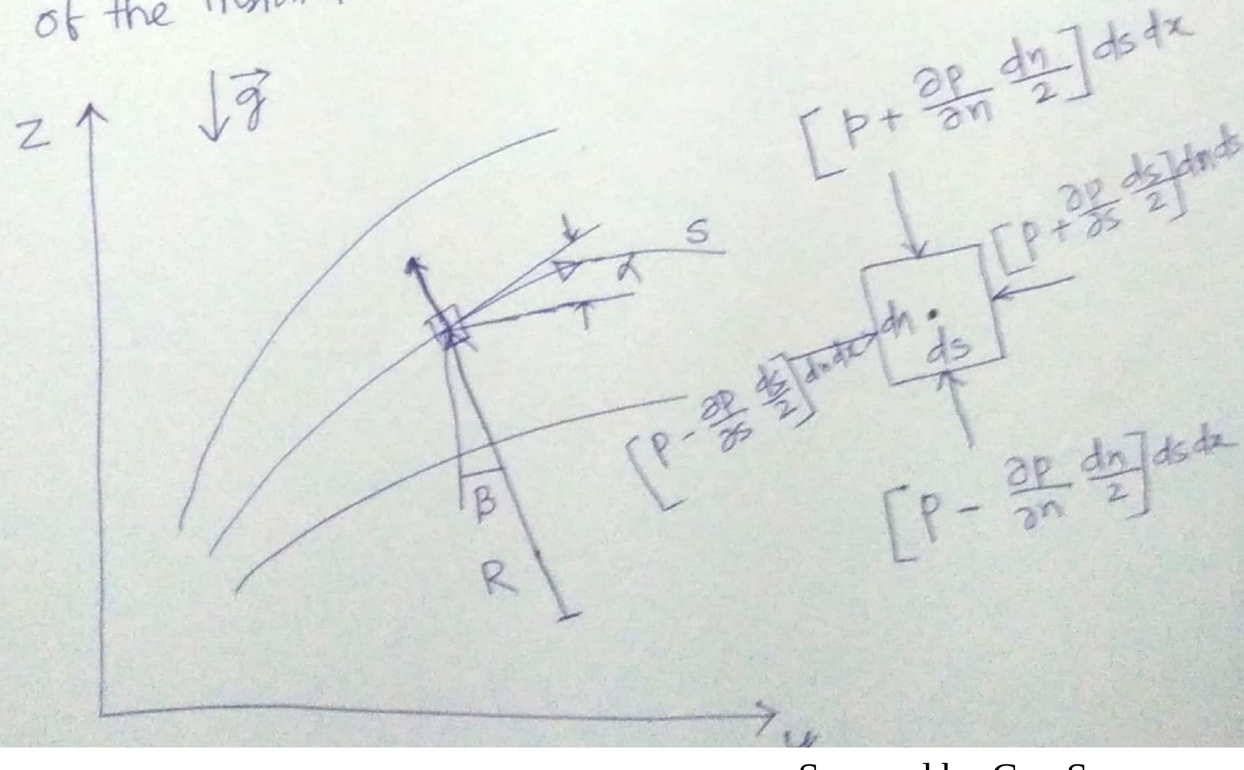
→ Streamlines, drawn tangent to the velocity vectors at every point in the flow field.

→ In a steady flow a fluid particle will move along a streamline because, for steady flow pathlines and streamlines coincide.

Pathline — line trace by a single fluid particle. (no of function of velocity → steady flow)

→ Thus in describing the motion of a fluid particle in a steady flow, in addition to using orthogonal co-ordinates x, y, z, the distance along a streamline is a logical co-ordinate to use in writing the equation of motion.

→ "streamline coordinates" also may be used to describe unsteady flow. streamlines in unsteady flow give a graphical representation of the instantaneous velocity field.



For simplicity,

→ Consider the flow in the yz plane as shown in fig.

→ We wish to write the equation of motion in terms of the co-ordinates s, distance along a streamline and the co-ordinate n, distance normal to the streamline.

→ The pressure at the centre of the fluid ~~particle~~ element is p.

→ If we apply newtons second law in the direction 's' of the streamline, to the fluid element of volume ds dn dx, then neglecting viscous ~~ete~~ forces we obtain.

$\frac{m}{V} = \rho$

$$\left(p - \frac{\partial p}{\partial s}\frac{ds}{2}\right) dn\,dx - \left(p + \frac{\partial p}{\partial s}\frac{ds}{2}\right) dn\,dx - \rho g \sin\beta \, ds\,dn\,dx = \rho a_s \, ds\,dn\,dx$$

where β is the angle between tangent to the streamline and
a_s is the acceleration of the fluid particle along the streamline

Simplifying the equation, we obtain

$$-\frac{\partial p}{\partial s} - \rho g \sin\beta = \rho a_s$$

Since $\beta = \frac{\partial z}{\partial s}$, we can write

$$-\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{\partial z}{\partial s} = a_s$$

(16)

Along any streamline $V = V(s,t)$, and the material or total acceleration of a fluid particle in the streamwise direction is given by

$$a_s = \frac{DV}{Dt} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$$

Euler's equation in the streamwise direction with the z axis directed vertically upward is then

$$-\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{\partial z}{\partial s} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} \quad \text{—(1)}$$

For steady flow, and neglecting the body forces, Euler's equation in the streamwise direction reduces to

$$\frac{1}{\rho}\frac{\partial p}{\partial s} = -V\frac{\partial V}{\partial s} \quad \text{——(2)}$$

which indicates that (for an incompressible, inviscid flow) — a decrease in velocity is accompanied by an increase in pressure and conversely.

→ The only force experienced by the particle is the net pressure force, so the particle accelerates towards ~~the~~ low pressure regions in the direction normal to the streamlines, we apply Newton's second law in the n-direction to the fluid element.

(7) Again neglecting viscous forces, we obtain

$$\left(p - \frac{\partial p}{\partial n}\frac{dn}{2}\right) ds\, dx - \left(p + \frac{\partial p}{\partial n}\frac{dn}{2}\right) ds\, dx$$

$$- \rho g \cos\beta \, dn\, dx\, ds = \rho a_n \, dn\, dx\, ds$$

$\beta \rightarrow$ in the angle between the n direction and the vertical, and a_n is the acceleration of the fluid particle in the n direction.

Simplifying the equation, we obtain.

$$-\frac{\partial p}{\partial n} - \rho g \cos\beta = \rho a_n$$

$$\cos\beta = \frac{\partial z}{\partial n}$$, we write.

$$-\frac{1}{\rho}\frac{\partial p}{\partial n} - g\frac{\partial z}{\partial z} = a_n$$

— The normal acceleration of the fluid element is towards the center of curvature of the streamline, in the minus n direction, thus in te co-ordinate system of fig 1 the familiar centripetal acceleration is written

$$a_n = -\frac{V^2}{R}$$

For steady flow, where R is te radius of curvature of the streamline at the point chosen.

(8)

~~For steady flow, we~~

Then, Euler's equation normal to the streamline is written for steady flow as

$$\frac{1}{\rho}\frac{\partial p}{\partial n} + g\frac{\partial z}{\partial n} = \frac{V^2}{R} \quad — (3)$$

For steady flow in a horizontal plane, Euler equation normal to ~~the~~ a streamline becomes

$$\frac{1}{\rho}\frac{\partial p}{\partial n} = \frac{V^2}{R} \quad —— (4)$$

→ pressure increases in the direction outward from the center of curvature of the streamlines

→ Because the only force experienced by the particle is the net pressure force, the pressure field creates the centripetal acceleration.

→ In region where the streamlines are straight, the radius of curvature R, is infinite so there is no pressure variation normal to the straight streamlines.

Bernoulli Equation : Integration of Euler's Equation Along a streamline for steady flow :-

→ Compaired to the viscous-flow equivalents, the momentum or Euler's equation for incompressible, inviscid flow is simpler mathematically, but solution still presents formidable difficulties in all but the most basic flow problems.

→ One convenient approach for a steady flow is to integrate Euler's equation along a streamline.

→ We will do this below using two different mathematical approaches and we will result in the Bernoulli equation.

(A) Derivation Using Streamline Coordinates

Euler's equation for steady flow along a streamline. is

$$-\frac{1}{\rho}\frac{\partial p}{\partial s} - g\frac{\partial z}{\partial s} = V\frac{\partial V}{\partial s} \quad \text{—} ①$$

If a fluid particle move a distance, ds, along a streamline, then

$\frac{\partial p}{\partial s} ds = dp$ (the change in pressure along 's')

$\frac{\partial z}{\partial s} ds = dz$ (the change in elevation along 's')

$\frac{\partial V}{\partial s} ds = dV$ (the change in speed along 's')

Thus after multiplying eqⁿ ① by ds, we can write

$$-\frac{dp}{\rho} - g\,dz = V\,dV$$

or $$\frac{dp}{\rho} + V\,dV + g\,dz = 0 \quad \text{(along s)}$$

Integration of this equation gives

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant} \quad \text{(along s)} \quad \text{— ②}$$

For the special case of incompressible flow, $\rho = \text{constant}$, Eqⁿ ② becomes the Bernoulli eqⁿ

$$\boxed{\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{Constant}} \quad \text{— ③}$$

Restrictions

1) Steady flow
2) Incompressible flow
3) Frictionless flow
4) Flow along a streamline

→ Bernoulli equation is probably the most famous equation in fluid mechanics.

→ It is always tempting to use because it is simple algebraic equation for relating the pressure, velocity, and elevation in a fluid.

→

(B)* Derivation Using Rectangular Co-ordinates :-

The vector form of Euler's equation

$$\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p$$

also can be integrated along a streamline.

→ We shall restrict the derivation to steady flow, thus, the end result of our effort shall be eqn ②.

For steady flow, Euler's equation in rectangular coordinates can be expressed as

$$\frac{D\vec{V}}{Dt} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

$$= (\vec{V}\cdot\nabla)\vec{V} = -\frac{1}{\rho}\nabla p - g\hat{k} \quad \text{— ⑤}$$

For steady flow the velocity field is given by

$$\vec{V} = \vec{V}(x, y, z).$$

The streamlines are line drawn in the flow field tangent to the velocity vector at every point. Recall again that for steady flow, streamlines, pathlines and streaklines coincide.

The motion of a particle along a streamline is governed by eqn ⑤

During time interval 'dt' the particle has vector displacement $\vec{ds}$ along the streamline.

If we take the dot product of the term in eq^n (5) with displacement '$\vec{ds}$' along the streamline, we obtain a scalar equation relating to pressure, speed and elevation along the streamline.

Taking the dot product of ~~equation~~ $\vec{ds}$ with eq^n (5) we have.

$$(\vec{V}\cdot\nabla)\vec{V}\cdot\vec{ds} = -\frac{1}{\rho}\nabla p\cdot\vec{ds} - g\hat{k}\cdot\vec{ds} \quad -(6)$$

where $\vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$ (along s)

Now evaluate each of the three terms in eq^n (6) starting on the right.

$$-\frac{1}{\rho}\nabla p\cdot\vec{ds} = -\frac{1}{\rho}\left[\hat{i}\frac{\partial p}{\partial x} + \hat{j}\frac{\partial p}{\partial y} + \hat{k}\frac{\partial p}{\partial z}\right]\cdot[*]$$

$$[*] = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$= -\frac{1}{\rho}\left[\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right] \text{ (along s)}$$

$$-\frac{1}{\rho}\nabla p\cdot\vec{ds} = -\frac{1}{\rho}dp \quad \text{(along s)}$$

and
$$-g\hat{k}\cdot\vec{ds} = -g\hat{k}\cdot[dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}]$$
$$= -g\,dz \quad \text{(along s)}$$

(5)

Using vector identity, we can write

$$(\vec{V}\cdot\nabla)\vec{V}\cdot ds = \left[\frac{1}{2}\nabla(\vec{V}\cdot\vec{V}) - \vec{V}\times(\nabla\times\vec{V})\right]\cdot ds$$

$$= \left\{\frac{1}{2}\nabla(\vec{V}\cdot\vec{V})\right\}\cdot\vec{ds} - \left\{\vec{V}\times(\nabla\times\vec{V})\right\}\cdot\vec{ds}$$

The last term on the right side of this equation is zero, since $\vec{V}$ is parallel to $\vec{ds}$

$$\left[\ \vec{V}\times(\nabla\times\vec{V})\cdot\vec{ds} = -(\nabla\times\vec{V})\times\vec{V}\cdot ds \right.$$
$$\left. = -(\nabla\times\vec{V})\cdot\vec{V}\times ds\ \right]$$

$$\therefore (\vec{V}\cdot\nabla)\vec{V}\cdot\vec{ds} = \frac{1}{2}\nabla(\vec{V}\cdot\vec{V})\cdot\vec{ds}$$

$$= \frac{1}{2}\nabla(V^2)\cdot\vec{ds} \text{ (along s)}$$

$$= \frac{1}{2}\left[\hat{i}\frac{\partial V^2}{\partial x} + \hat{j}\frac{\partial V^2}{\partial y} + \hat{k}\frac{\partial V^2}{\partial z}\right]\cdot[*]$$

$$[*] = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$= \frac{1}{2}\left[\frac{\partial V^2}{\partial x}dx + \frac{\partial V^2}{\partial y}dz + \frac{\partial V^2}{\partial z}dz\right]$$

$$(\vec{V}\cdot\nabla)\vec{V}\cdot\vec{ds} = \frac{1}{2}d(V^2) \text{ (along s)}$$

Substituting these three terms in eq^n (6)

$$\frac{dp}{\rho} + \frac{1}{2} d(V^2) + g\,dz = 0 \quad (\text{along } s)$$

Integrating this equation, we obtain

$$\int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant} \quad (\text{along } s)$$

If the density is constant, we obtain the Bernoulli equation

$$\boxed{\frac{p}{\rho} + \frac{V^2}{2} + gz = \text{constant}}$$

The Bernoulli equation derived using rectangular co-ordinates is still subject to restrictions :

(1) steady flow
(2) incompressible flow
(3) frictionless flow
and (4) flow along a streamline

(1)

Static, Stagnation, and Dynamic Pressure

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = C$$

→ The pressure, P, which we use in deriving the Bernoulli equation is thermodynamic pressure. it is most commonly called the static Pressure.

→ The static pressure is the pressure experienced by the fluid particle as it move.

→ We also have stagnation and dynamic pressure.

→ Stagnation pressure is obtained when a flowing fluid is decelerated to zero speed by a frictionless process.

For incompressible flow, the Bernoulli equation can be used to relate changes in speed and pressure along a streamline for such a process.

$$\frac{P}{\rho} + \frac{V^2}{2} = \text{constant} \quad -(1)$$

If the static pressure is p at a point in the flow where the speed is V, then the stagnation pressure, P_0, where the stagnation speed, V_0 is zero, may be computed from.

$$\frac{P_0}{\rho} + \frac{V_0^2}{2} \overset{\nearrow 0}{} = \frac{P}{\rho} + \frac{V^2}{2} \quad -(2)$$

$$P_0 = P + \frac{1}{2} \rho V^2 \quad \text{— (3)}$$

Eqⁿ (3) is mathematical statement of the definition of stagnation pressure valid for incompressible flow.

The term $\frac{1}{2}\rho V^2$ generally is called the dynamic pressure.

Eqⁿ (3) states that the stagnation pressure (total pressure) equals the static pressure plus the dynamic pressure.

⇒ One way to picture the three pressure is to imagine you are standing in a steady wind holding up your hand:
The static pressure will be atmospheric pressure; the larger pressure you feel at the center of your hand will be the stagnation pressure and the buildup of stagnation pressure (the difference between stagnation and static pressures) will be the dynamic pressure.

Eqⁿ (3) gives.

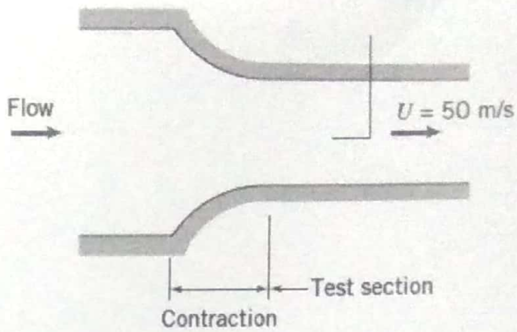
$$V = \sqrt{\frac{2(P_0 - P)}{\rho}} \quad \text{— (4)}$$

If the P_0 & P is measured at a point Eqⁿ 4 will give the local flow speed.

(3)

* Bernoulli equation applies only for incompressible flow (Mach number M ≤ 0.3).

6.44 The inlet contraction and test section of a laboratory wind tunnel are shown. The air speed in the test section is $U = 50$ m/s. A total-head tube pointed upstream indicates that the stagnation pressure on the test section centerline is 10 mm of water below atmospheric. The laboratory is maintained at atmospheric pressure and a temperature of −5°C. Evaluate the dynamic pressure on the centerline of the wind tunnel test section. Compute the static pressure at the same point. Qualitatively compare the static pressure at the tunnel wall with that at the centerline. Explain why the two may not be identical.



Given: Wind tunnel with inlet section

Find: Dynamic and static pressures on centerline; compare with Speed of air at two locations

Solution:

Basic equations $p_{dyn} = \frac{1}{2}\cdot\rho_{air}\cdot U^2$ $\quad p_0 = p_s + p_{dyn}$ $\quad \rho_{air} = \frac{p}{R_{air}\cdot T}$ $\quad \Delta p = \rho_w\cdot g\cdot\Delta h$

Assumptions: 1) Incompressible flow 2) Inviscid 3) Steady 4) Along a streamline

Available data $T = -5\,°C$ $\quad U = 5($ $R = 287\cdot\frac{J}{kg\cdot K}$ $\quad p_{atm} = 101\cdot kPa$ $\quad h_0 = -10\cdot mm$ $\quad \rho_w = 999\cdot\frac{kg}{m^3}$

For air $\rho_{air} = \frac{p_{atm}}{R\cdot T}$ $\quad \rho_{air} = 1.31\frac{kg}{m^3}$

$p_{dyn} = \frac{1}{2}\cdot\rho_{air}\cdot U^2$ $\quad p_{dyn} = 1.64\cdot kPa$

Also $p_0 = \rho_w\cdot g\cdot h_0$ $\quad p_0 = -98.0\,Pa$ (gage)

and $p_0 = p_s + p_{dyn}$ so $p_s = p_0 - p_{dyn}$ $\quad p_s = -1.738\,kPa$ (gage) $\quad h_s = \frac{p_s}{\rho_w\cdot g}$ $\quad h_s = -177\,mm$

Streamlines in the test section are straight so $\frac{\partial}{\partial n}p = 0$ and $p_w = p_{centerline}$

In the curved section $\frac{\partial}{\partial n}p = \rho_{air}\cdot\frac{V^2}{R}$ so $p_w < p_{centerline}$

Module IV

Flow through Pipes (Incompressible Flow)

Energy losses in pipe flow

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of the fluid is lost.

The loss of energy is classified as

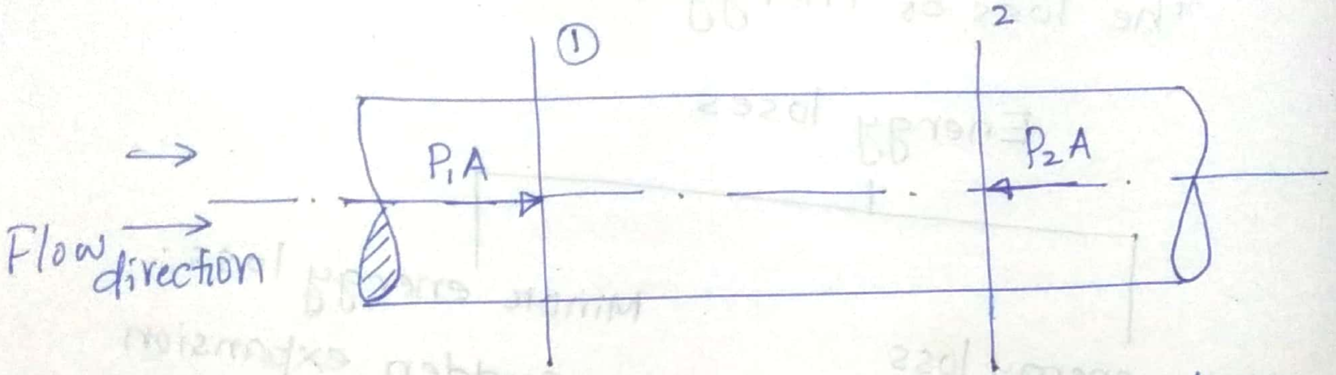
Energy loses

- Major energy loss (due to friction)
- Minor energy loss:
  - (a) sudden expansion
  - (b) sudden contraction
  - (c) bend in pipe
  - (d) pipe fitting
  - (e) An obstruction in pipe.

Frictional loses in pipe flows :

* The viscosity causes loss of energy in flows which is known as frictional loss.

Expression for loss of head



Consider a horizontal pipe, having steady flow as shown above.

Let L = Length of the pipe between section 1 and 2

d = diameter of the pipe

f' = ~~friction factor~~ / frictional resistance per unit wetted area per unit velocity.

h_f = loss of head due to friction

P_1 = pressure at section 1

V_1 = velocity at section 1

P_2, V_2 are corresponding values at section 2

Applying Bernoulli's equations for real fluid at section 1 and 2, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2}{2g} + Z_2 + h_f$$

But $Z_1 = Z_2$ and $V_1 = V_2$, as the pipe is horizontal and diameter of the pipe is same in both sections.

$$h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \quad \text{— (1)}$$

Now, frictional resistance ~~per unit area~~ = frictional resistance per unit wetted area per unit velocity × wetted area × velocity²

$$F_1 = f' \times (PL) \times V^2$$

Here P = ~~per~~ wetted perimeter

For equilibrium in the direction of flow, resolving in the horizontal direction, we have ~~add~~ all the forces

$$P_1 A - P_2 A - F_1 = 0$$

$$P_1 A - P_2 A = F_1 = f' \times PL \times V^2$$

$$\Rightarrow (P_1 - P_2) A = \frac{f' \times PL \times V^2}{A} \quad \text{— (2)}$$

$$\Rightarrow P_1 - P_2 =$$

From eqⁿ (1) $P_1 - P_2 = \rho g h_f$ —— (3)

Equating eq^n (2) and (3),

$$\rho g h_f = \frac{f' \times PL \times V^2}{A} \quad \text{— (3a)}$$

$$\frac{\pi D L}{\frac{\pi}{4} D^2} = \frac{4L}{D}$$

$$\Rightarrow h_f = \frac{f'}{\rho g} \times \frac{4 L V^2}{d} \quad \text{— (4)}$$

Substituting $\frac{f'}{\rho g} = \frac{f}{2g}$ $f \to$ known as co-efficient of friction.

Equation (4) becomes

$$h_f = \frac{f}{2g} \times \frac{4 L V^2}{d}$$

$$\boxed{h_f = \frac{4 f L V^2}{2gd}} \quad \text{— (5)}$$

f is known as co-efficient of friction ~~factor~~, which is dimensionless quantity.

Equation (5) is Darcy – Weisbach equation

f = Co-efficient of friction which is a function of Reynold number.

$$f = \frac{16}{R_e} \quad \text{for } R_e < 2000 \text{ (viscous flow)}$$

$$= \frac{0.079}{R_e^{1/4}} \quad \text{for } R_e \text{ varying from 4000 to } 10^6.$$

~~Re 4000 < Re < 10^6~~

$L \to$ length of pipe $V \to$ mean velocity of flow
$d \to$ diameter of pipe.

$$h_f = \frac{4fLV^2}{2gd} \quad \text{— (5)}$$

$f \rightarrow$ co-efficient of friction

Equation (5) is re-written as

$$h_f = \frac{fLV^2}{2gd}$$

Here f is know as <u>friction factor</u> and it is dimensionless.

It may be noted that friction factor is four times the co-efficient of friction.

(*) For laminar flow, f is a function of the Reynolds number only, (i.e)

$$f = \frac{64}{Re}, \quad R_e < 2000 \qquad \text{Hagen – Poiseuille}$$

$$f = \frac{0.316}{R_e^{1/4}}, \quad R_e < 10^5 \qquad \text{Blasius}$$

(*) For a fully developed turbulent flow, f is independent of Reynolds number, and it is a function of relative roughness (ϵ/d) alone, where ϵ is the <u>roughness projection</u>.

(*) For the transition region, f depends on both the Reynolds number and the relative ~~rough~~ roughness, (i.e) $f = \xi\left(R_e, \frac{\epsilon}{d}\right)$

(*) For commercial pipes, the friction factor f as a function of Re and $\frac{\epsilon}{d}$ is plotted on a logarithmic paper called ~~Mod~~ Moody's chart.

The chart can also be used for non-circular sections and Re can be calculated by replacing d by 4R, where $R = \frac{A}{P}$

$R_e = \frac{\rho v}{\mu}$

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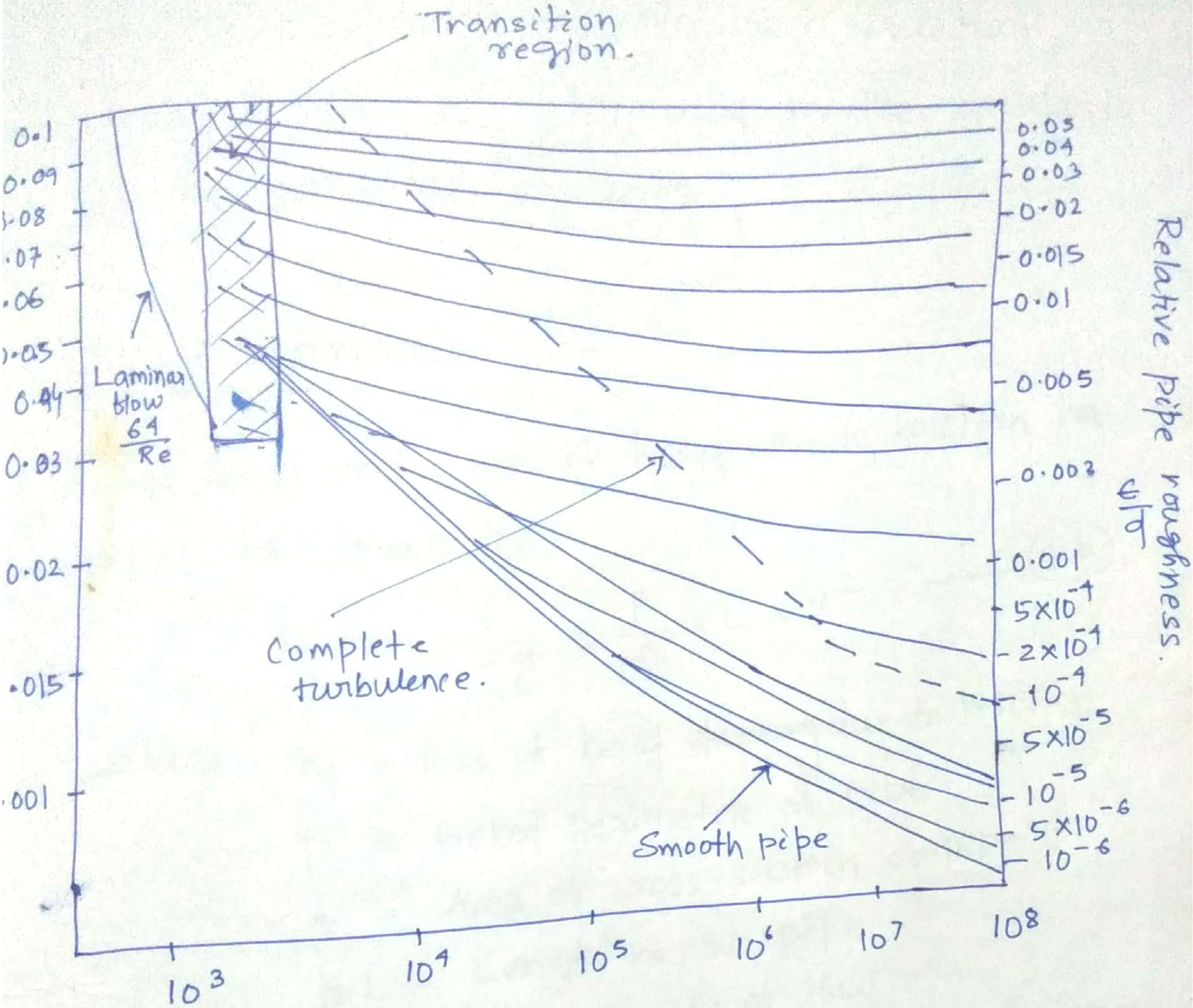
$f = \xi(Re, \epsilon/d)$ → logarithmic paper called



$$Re = \frac{\rho v d}{\mu}$$

Reynolds number.

<u>Moody's chart</u>

ϵ ⟷ Materials. ⟶ ϵ

Glass — .0025

Cast iron. — 0.15

steel — 0.025

In addition to Darcy-Weisbach equation, chezy's formula also used for the analysis of the pipe flow problems.

Chezy's formula :-

We know that loss of head due to friction in pipe is given by. — (3b)

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$$

where, h_f = loss of head ~~during~~ due to friction

P = wetted perimeter of pipe

A = Area of cross-section of pipe

L = Length of the pipe

V = mean velocity of flow

Now, the ratio $\frac{A}{P}$ $\left(= \frac{\text{Area of flow}}{\text{wetted perimeter}}\right)$ is called hydraulic radius and hydraulic mean depth or hydraulic radius and is denoted by m.

∴ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

∴ $\frac{P}{A} = \frac{1}{m}$ in eqn (3b)

Substituting

$$h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times V^2$$

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$$h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times V^2$$

$$\Rightarrow V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$= \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\Rightarrow V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}}$$

$$= \sqrt{\frac{\rho g}{f'}} \times \sqrt{m \frac{h_f}{L}} \quad \text{—— (6)}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$

where C is a constant known as Chezy's constant

and $\frac{h_f}{L} = i$ where i → is loss of head per unit length of pipe

substituting the values of $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$, we get

$$\boxed{V = C\sqrt{mi}} \quad \text{—— (7)}$$

Equation (7) is known as Chezy's formula

Hydraulic gradient and total energy line :-

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes.

Hydraulic gradient line : It is defined as the line which gives the sum of pressure head $\left(\frac{P}{\rho g}\right)$ and datum/potential head (z) of a flowing fluid in a pipe with respect to some reference line.

or,

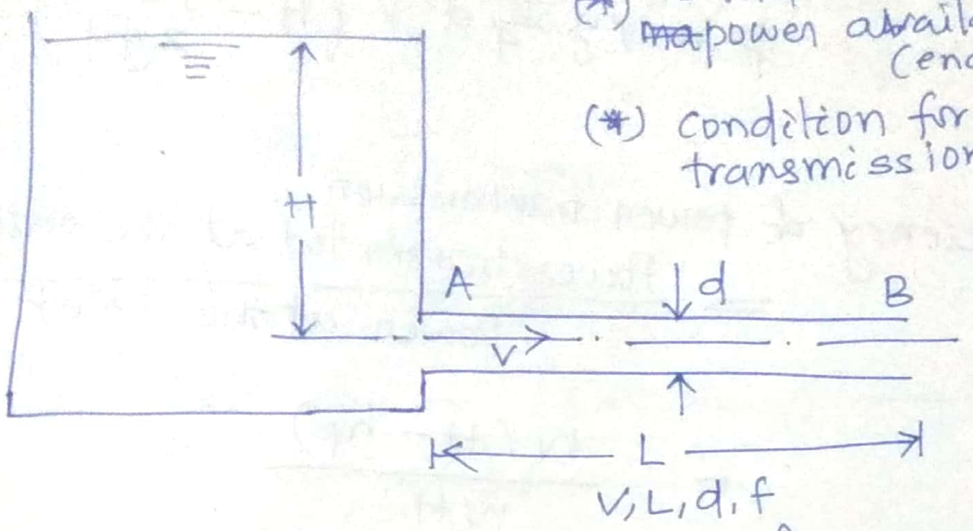
It is the line which is obtained by joining the top of all vertical ordinates showing the pressure head $\left(\frac{P}{\rho g}\right)$ of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient line)

Total Energy Line : It is defined as the line which gives the sum of pressure head $\left(\frac{P}{\rho g}\right)$, datum/potential head (z) and kinetic head $\left(\frac{V^2}{2g}\right)$ of a flowing fluid in a pipe with respect to some reference line.

It is also defined as the line which is obtained by joining the top of all vertical ordinates showing the sum of pressure head $\left(\frac{P}{\rho g}\right)$ and kinetic head $\left(\frac{V^2}{2g}\right)$ from the centre of the pipe. It is briefly written as T.E.L. (Total energy line)

Power transmission through Pipes

(a) * Consider a pipe AB connected to a tank.



(*) To find power available at B (end of pipe)

(*) condition for maximum transmission of power.

(*) Power is transmitted through pipe by flowing liquid through them

(*) Power transmitted depends upon
- (*) Weight of the liquid flowing through the pipe.
- (*) total head available at the end of pipe

Let d = diameter of the pipe
V = Velocity of flow in pipe
H = total head available at the inlet
h_f = head loss due to friction
f = friction factor.

Total head available at the outlet

$$= H - h_f = H - \frac{fLV^2}{2gd}$$

minor losses are neglected.

Total weight of liquid flowing through pipe per sec $W = \rho g \frac{\pi}{4} d^2 V$

$W = \rho g \times$ volume of water per sec.

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$(energy/sec) = \frac{weight}{sec} \times$ head at outlet

The power transmitted at the outlet of the pipe

$$p = \rho g \frac{\pi}{4} d^2 V \left(H - \frac{fLV^2}{2gd} \right) \quad \text{— (1)}$$

Efficiency of power transmission

$$= \frac{\text{Power transmitted at the outlet}}{\text{Power at the inlet}}$$

$$= \frac{W(H - h_f)}{WH}$$

$$= \frac{H - h_f}{H} \quad \text{— (2)}$$

Maximum transmission of power

$$\frac{dp}{dV} = 0$$

$$\Rightarrow \rho g \frac{\pi}{4} d^2 \left(H - \frac{3fLV^2}{2gd} \right) = 0$$

$$\Rightarrow H - \frac{3fLV^2}{2gd} = 0$$

$$\Rightarrow H - 3h_f = 0 \Rightarrow H = 3h_f$$

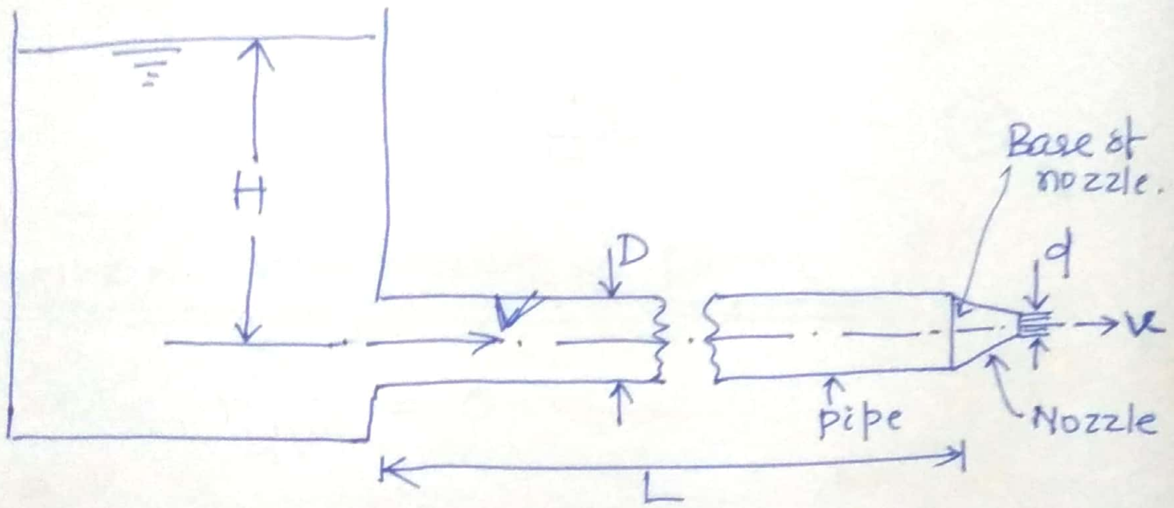
$$\Rightarrow \boxed{h_f = \frac{H}{3}} \quad \text{— (3)}$$

Head available is 3 times the loss of head due to friction.

Flow through nozzles

(*) A nozzle is a gradually converging short tube

(*) Fitted at the outlet ~~of a long pi~~ end of a pipe for the purpose of converting the total energy of the flowing water into velocity (kinetic) energy.



Let D = diameter of pipe

A = Area of the pipe.

V = Velocity of flow in pipe

H = total head at the inlet of the pipe

d = diameter of nozzle at outlet

v = velocity of flow at outlet of nozzle

a = area of nozzle at outlet

f = friction factor

Head available at the end of the pipe / base of nozzle

$$= H - h_f$$

$$= H - \frac{fLV^2}{2gD}$$ Neglecting minor losses and losses in the nozzle.

Total head at the inlet of the pipe = Total head at the outlet of nozzle + losses.

$$\therefore H = \frac{v^2}{2g} + h_f$$

$$= \frac{v^2}{2g} + \frac{fLV^2}{2gD} \quad \text{—— (1)}$$

From continuity equation, in the pipe and outlet of nozzle

$$AV = av$$

$$\Rightarrow V = \frac{av}{A} \quad \text{—— (2)}$$

Substituting the value of V in eqn (1)

$$\therefore H = \frac{v^2}{2g} + \frac{fL}{D}\frac{a^2}{A^2}\frac{v^2}{2g}$$

$$= \frac{v^2}{2g}\left(1 + \frac{fL}{D}\frac{a^2}{A^2}\right)$$

$$\Rightarrow v = \sqrt{\frac{2gH}{1 + \frac{fL}{D}\frac{a^2}{A^2}}} \quad \text{—— (3)}$$

Kinetic energy of the jet at the outlet per sec

$$= \frac{1}{2} m v^2 = \frac{1}{2} (\rho a v) v^2$$

$$= \frac{1}{2} \rho a v^3$$

Efficiency of power transmission.

$$\eta = \frac{\text{Head transmitted}}{\text{head supplied}}$$

$$= \frac{\frac{v^2}{2g}}{H}$$

$$= \frac{v^2}{2gH} = \frac{1}{1 + \frac{fL}{D} \cdot \frac{a^2}{A^2}}$$

Maximum power available from a nozzle

Power transmitted through nozzle

✓ $= ~~\frac{1}{2} \rho a v^3~~$ ~~$\frac{4}{2}$~~ ~~$\frac{v^2}{g}$~~

✓ $= ~~\frac{4}{4} \rho a v~~ \frac{v^2}{2}$

$$= \rho g a v \cdot \frac{v^2}{2g}$$

Substituting the value of $\frac{v^2}{2g}$ from equation 1

$$= \rho g a v \left[H - \frac{fLV^2}{2gD} \right]$$

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substituting the value of V from eqn (2), we get

$$P = \rho g a v \left[H - \frac{fL}{D} \frac{a^2}{A^2} \frac{v^2}{2g} \right]$$

$$= \rho g a \left[H v - \frac{fL}{D} \frac{a^2}{A^2} \frac{v^3}{2g} \right]$$

Condition for maximum power transmitted by nozzle

$$\frac{dp}{dv} = 0$$

$$\Rightarrow \rho g a \left[H - 3 \frac{fL}{D} \frac{a^2}{A^2} \frac{v^2}{2g} \right] = 0$$

$$\Rightarrow H - \frac{3fL}{D} \frac{a^2}{A^2} \frac{v^2}{2g} = 0$$

$$\Rightarrow H = \frac{3fL}{D} \frac{V^2}{2g}$$

$$\boxed{H = 3h_f} \quad \text{--- (4)}$$

∴ power ~~transmission~~ transmitted by a nozzle is maximum when the head supplied is equal to three times the loss of head due to friction in the pipe.

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Diameter of the nozzle for transmitting maximum power

For maximum transmission of power

$$H = 3 h_f$$

$$= 3 \frac{fLV^2}{2gD}$$

Substituting in eq^n (1)

$$H = \frac{v^2}{2g} + \frac{fLV^2}{2gD}$$

$$\Rightarrow \frac{3fLV^2}{2gD} = \frac{v^2}{2g} + \frac{fLV^2}{2gD}$$

$$\Rightarrow \frac{2fLV^2}{2gD} = \frac{v^2}{2g}$$

$$\Rightarrow \frac{fLV^2}{2gD} = \frac{1}{2} \times \frac{v^2}{2g}$$

Since $V = \frac{av}{A}$

$$\frac{fL \frac{a^2}{A^2} v^2}{2gD} = \frac{1}{2} \times \frac{v^2}{2g}$$

$$\Rightarrow \frac{2fL}{D} = \frac{A^2}{a^2} \Rightarrow$$

$$\Rightarrow \boxed{\frac{A}{a} = \sqrt{\frac{2fL}{D}}} \quad \text{——— (5)}$$

Eq^n (5) gives the ratio between area of supply pipe and no[zzle] for maximum po[wer] tran[smission]

Measurements

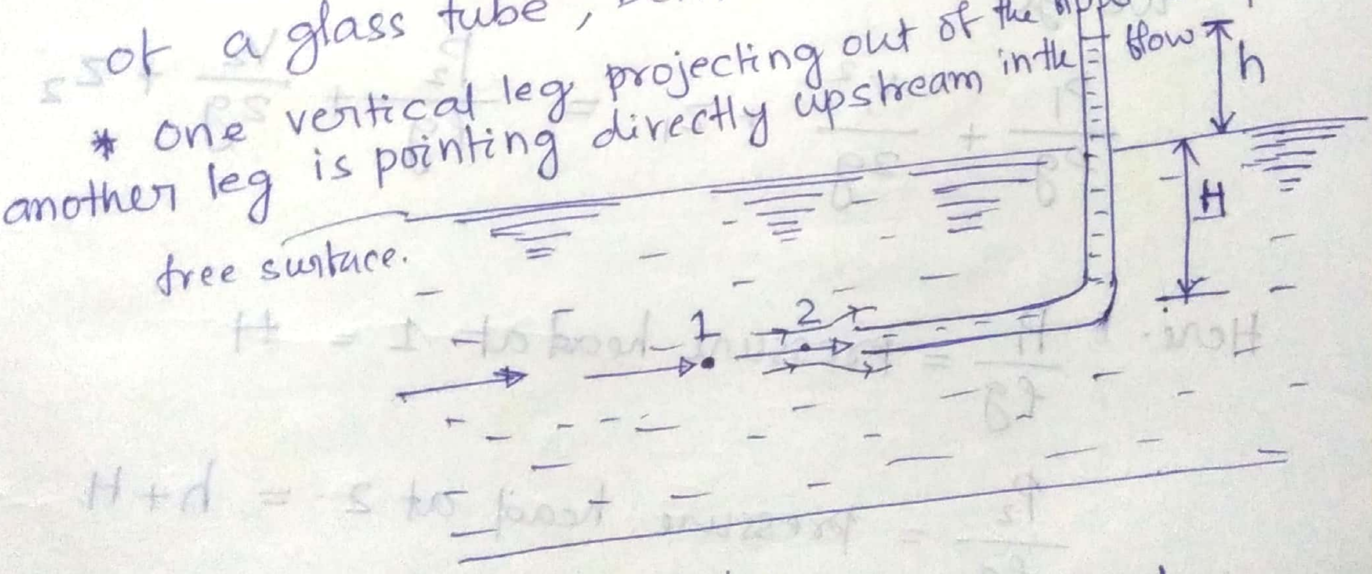
Pitot - tube (Henri de Pitot) originator

* It is a device used for measuring the velocity of flow at any point in a pipe or channel.

* It is based on the principle that : If the velocity at any point decreases, the pressure at that point increases due to the ~~conver~~ ~~conservation of~~ conversion of kinetic energy into pressure energy.

* In simplest form, the pitot tube consists of a glass tube, bent at right angles.

* one vertical leg projecting out of the flow and another leg is pointing directly upstream in the flow



Pitot tube

point 1 and 2 at the same level.

point 2 is just at the inlet of the pitot tube

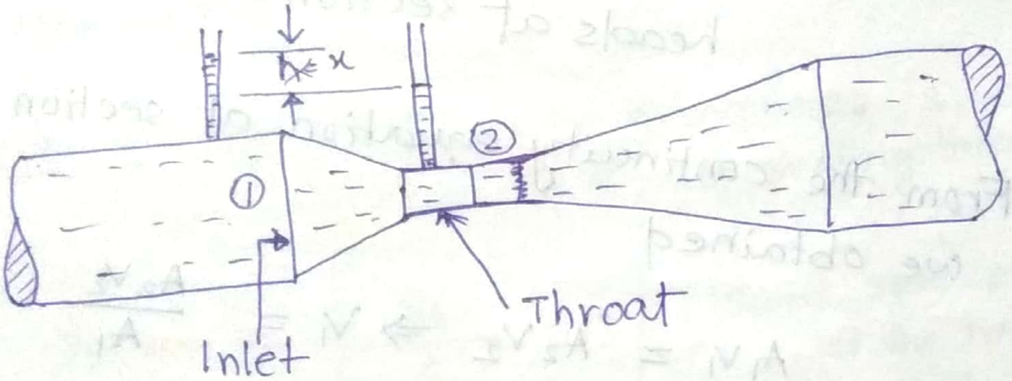
point 1 is far away from the tube

Venturimeter :

Venturimeter is a device used for measuring the rate of flow of fluid flowing through a pipe.

It consists of three parts.

- A short converging part
- Throat
- Diverging part



Let
d_1 = diameter at the inlet (section 1)
p_1 = pressure at section 1
v_1 = velocity at section 1
A_1 = Area at section 1

d_2, P_2, V_2, A_2 are the corresponding values at throat (section 2)

Applying Bernoulli's equation at section 1 and 2, we get,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

1

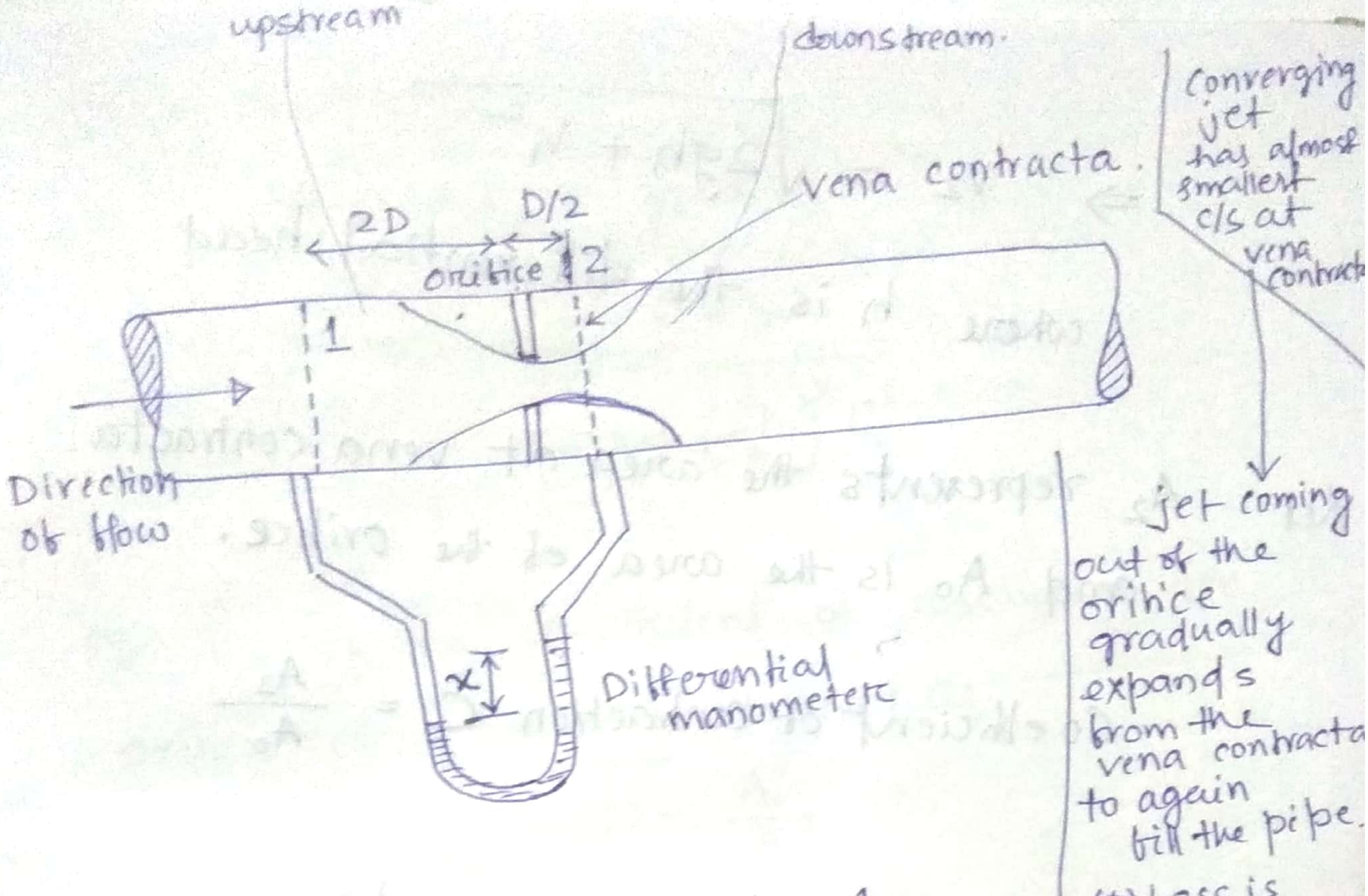
Orifice Meter

* It is a device used for measuring the rate of flow of fluid flowing through a pipe.

* It is a cheaper device as compaired to venturimeter. This also work on the same principle as that of the venturimeter.

* It consists of flat circular plate which has a circular hole in concentric with the pipe. This is called orifice.

* The diameter of the orifice is generally 0.5 times the diameter of the pipe although it may very from 0.4 to 0.8 times the pipe diameter.



Let d_1 = diameter at section 1

P_1 = pressure at section 1

V_1 = velocity at section 1

A_1 = area at section 1

d_2, P_2, V_2 and A_2 are the corresponding values at section 2.

Applying Bernoulli's equation at section 1 and 2, we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{V_2^2 - V_1^2}{2g}$$

$$\Rightarrow h = \frac{V_2^2 - V_1^2}{2g}$$

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